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The Consumption Euler Equation or the Keynesian Consumption Function?*

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Abstract

We formulate a general cointegrated vector autoregressive (CVAR) model that nests both a class of consumption Euler equations and various Keynesian-type consumption functions. Using likelihood-based methods and Norwegian data, we find support for cointegration between consumption, income and wealth once a structural break around the time of the financial crisis is allowed for. The fact that consumption cointegrates with both income and wealth and not only with income points to the empirical irrelevance of an Euler equation. Moreover, we find that consumption equilibrium corrects to changes in income and wealth, but not that income equilibrium corrects to changes in consumption, which would follow from an Euler equation. We also find that most of the parameters stemming from the class of Euler equations are not corroborated by the data when conditional expectations of future consumption and income in CVAR models are considered. Only habit formation seems important in explaining Norwegian consumer behaviour. Our estimated conditional Keynesian-type consumption function implies a first year marginal propensity to consume (MPC) out of income of close to 40%.

I. Introduction

Economists have long been concerned with how households react to changes in fiscal policy. The financial crisis in 2008 led to renewed interest in how household asset composition, liquidity and credit market conditions may affect consumption; see for instance Muellbauer (2016) and Kaplan *et al.* (2018). The effects of fiscal policy depend on the marginal propensity to consume (MPC) out of shocks to income. A new consensus seems

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to be emerging on the size of the MPC that is much larger than what used to be common in many DSGE models. For instance, the heterogeneity-augmented model by Carroll *et al.* (2017) predicts an aggregate MPC of around 20% compared to roughly 5% implied by macroeconomic models with representative agents.

In contrast to the Keynesian consumption function, which maintains that changes in current household income affect consumption markedly, both the permanent income hypothesis of Friedman (1957) and the life-cycle hypothesis of Ando and Modigliano (1963) imply that consumption depends on unanticipated and not on anticipated income shocks with a much stronger response to permanent than transitory shocks. These hypotheses are typically formulated as consumption Euler equations, where consumption of a representative agent does not respond much to transitory income changes. However, consumption Euler equations have found little support in aggregate data; see Flavin (1981), Campbell and Deaton (1989), Muellbauer and Lattimore (1995), Yogo (2004), Palumbo *et al.* (2006) and Canzoneri *et al.* (2007). Recent microeconometric studies also find that households react much more strongly to transitory income shocks than is predicted by the standard forward-looking theory of consumption. For instance, Jappelli and Pistaferri (2014) estimate an average MPC of 48% using Italian data, and Fagereng *et al.* (2019) find an a MPC that ranges between 35 and 70% using Norwegian data.

Extended versions of the standard forward-looking theory that allow for precautionary saving, liquidity constraints and habit formation can explain some of the empirical results found in the literature. Campbell and Mankiw (1991) among others account for precautionary saving and liquidity constraints in a model for aggregate consumption assuming constant relative risk aversion (CRRA) utility preferences and that some of the households are current income consumers. Deaton (1991) explains consumer behaviour by means of the so-called buffer-stock model, in which households facing liquidity constraints use liquid assets to buffer against temporary income shocks. Kaplan and Violante (2014) introduce trading costs to explain evidence of current income consumers even for those who are wealthy due to illiquid assets and credit constraints. The consumption model of Smets and Wouters (2003), which many DSGE models are typically based upon, includes habit formation in that current consumption is proportional to past consumption.

The contributions of the present paper are threefold. First, we formulate a general cointegrated vector autoregressive (CVAR) model that nests both a class of consumption Euler equations and various Keynesian-type consumption functions. The former includes a version of the martingale hypothesis of Hall (1978) and the precautionary saving and liquidity constraints equations as in Campbell and Mankiw (1991) and of habit formation as in Smets and Wouters (2003). Using likelihood methods, one can test the properties of *cointegration* between consumption and income and of *equilibrium correction* in the nested CVAR. Drawing upon Eitrheim *et al.* (2002), the former property represents the common ground for a Keynesian-type consumption function and a consumption Euler equation while the latter represents the discriminating feature between them.[1] The joint implication of a consumption Euler equation and existence of cointegration between consumption and income is that saving today predicts income declines tomorrow, the so-called ‘saving for a rainy day’ hypothesis of Campbell (1987).

[1] See also Anundsen and Nymoen (2019) for a recent application to American data.

Second, we study aggregate Norwegian consumer behaviour within the context of the nested CVAR using seasonally unadjusted quarterly data that span the period from the early 1980s to the end of 2016. We find support for cointegration between consumption, income and wealth once a structural break around the time of the financial crisis in 2008 is allowed for. Our finding that consumption cointegrates with both income and wealth and not only with income is evidence against a consumption Euler equation. Likelihood ratio tests also show that consumption equilibrium corrects to changes in income and wealth, but not that income equilibrium corrects to changes in consumption, as would be the case if an Euler equation were true. Our estimated conditional Keynesian-type consumption function implies a first year MPC of around 40%, which is in line with recent microeconometric evidence.

Third, we consider conditional expectations of future consumption and income in CVAR models within the context of Johansen and Swensen (1999, 2008). Since, as pointed out by Tinsley (2002), 'empirical rejection of rational expectations is the rule rather than the exception in macroeconomics', we divide the parameters of well-fitted CVAR models into two parts: parameters of interest, which are the parameters describing rational expectations, and nuisance parameters, which are the parameters necessary to ensure satisfactory empirical fit. Using this strategy, it is possible to focus on economically interesting parameters stemming from the class of Euler equations. Our treatment of the role of conditional expectations of future consumption and income is quite similar to what has been done in the new Keynesian literature on pricing behaviour; see Boug *et al.* (2010, 2017). We find that most of the parameters stemming from the class of Euler equations are not corroborated by the data when conditional expectations of future consumption and income in CVAR models are considered. Only habit formation in accordance with Smets and Wouters (2003) seems to play an important role in explaining Norwegian consumer behaviour.

The rest of the paper is structured as follows: Section II discusses the theoretical background and how the various hypotheses regarding consumer behaviour are nested within a general CVAR. Section III presents the data used in the empirical analysis. Section IV reports findings from the cointegration analysis and the estimation of consumption short-run dynamics. Section V presents the results of considering conditional expectations of consumption and income in CVAR models. Section VI provides a conclusion.

II. Theoretical background

As a useful benchmark for the empirical analysis, we first outline the martingale hypothesis and the 'saving for a rainy day' hypothesis. Then, we present the consumption Euler equations with precautionary saving, liquidity constraints and habit formation based on CRRA utility preferences. Finally, we formulate a CVAR that nests the various hypotheses from the set of Euler equations as well as Keynesian-type consumption functions.

The martingale and the 'saving for a rainy day' hypotheses

The main idea behind the permanent income hypothesis and the life-cycle hypothesis, which both the martingale and the 'saving for a rainy day' hypotheses build upon, is that aggregate consumption can be modelled as the intertemporal optimization decision under uncertainty by a representative consumer.

If the utility function of the consumer is quadratic and the riskless rate of real return is constant and equal to the subjective discount rate, the martingale hypothesis of Hall (1978) can be formulated as

$$E_t C_{t+1} = C_t, \tag{1}$$

where E_t and C_{t+1} denote expectations conditional on information at time t and consumption at time $t+1$ respectively. According to (1), no other variable than consumption at time t should help predict consumption at time $t+1$. This in turn implies that $\Delta C_{t+1} = \varepsilon_{t+1}$, where ε_{t+1} is an unforecastable innovation in permanent income. The change in consumption is thus unforecastable.

An alternative formulation of the permanent income hypothesis is the 'saving for a rainy day' hypothesis of Campbell (1987). As shown by Campbell and Deaton (1989) and used by Palumbo *et al.* (2006) among others, a logarithmic version of this hypothesis can be written as $\frac{S_t}{YL_t} \approx -\sum_{i=1}^{\infty} \rho^i E_t \Delta yl_{t+i} + \varsigma$, where $S_t \equiv Y_t - C_t$, $Y_t \equiv R(1+R)^{-1} W_t + YL_t$, R denotes the riskless rate of real return, W_t is financial wealth at time t, YL_t is labour income at time t, ρ denotes a discount factor and ς is a constant.[2] Hence, the saving ratio, $\frac{S_t}{YL_t}$, and expected future labour income growth, $E_t \Delta yl_{t+i}$, are negatively related so that saving increases today when the consumer anticipates that income will decline tomorrow. The consumer 'saves for a rainy day'.

We follow Eitrheim *et al.* (2002) and Anundsen and Nymoen (2019) and formulate the 'saving for a rainy day' hypothesis as

$$y_t - c_t \approx -\sum_{i=1}^{\infty} \rho^i E_t \Delta y_{t+i} + \varsigma, \tag{2}$$

where the saving ratio, $\frac{S_t}{YL_t}$, is approximated by the logarithms of income to consumption ratio, $y_t - c_t$, and labour income, yl_{t+i}, is replaced by income, y_{t+i}. An important time series property, which we shall utilize in the nested CVAR, is that the saving ratio is stationary, $I(0)$, and thus that income and consumption are cointegrated with a coefficient equal to one when income is non-stationary, $I(1)$.

Euler equations with CRRA preferences

To allow for precautionary saving, liquidity constraints and habit formation, we now turn to consumption Euler equations with CRRA preferences. Campbell and Mankiw (1991) apply the utility function $U(C_t) = (1-\delta)^{-1} C_t^{1-\delta}$ for $1 \neq \delta > 0$, where δ is the inverse of the intertemporal elasticity of substitution, σ. Assuming that the logarithms of consumption are normally distributed with mean $E_t \ln C_{t+1}$ and time-varying variance η_{t+1}^2, we can write the consumption Euler equation as

$$E_t \Delta c_{t+1} = \frac{\eta_{t+1}^2}{2\sigma} - \sigma\theta + \sigma R_t, \tag{3}$$

[2] Here and in the following, lower case letters denote the logarithms of a variable.

where θ and R_t denote the subjective discount rate, assumed constant, and the real interest rate, assumed *ex post* respectively. If the consumer faces more uncertainty, that is a larger η^2_{t+1}, consumption is expected to increase from this period to the next. Thus, the consumer reduces consumption now in response to increased uncertainty to have a larger safety buffer, that is precautionary saving, for more consumption in the next period. According to (3), saving by the consumer is also associated with intertemporal substitution in consumption. An increase in the real interest rate makes saving more profitable due to relatively costly consumption today; hence consumption is expected to increase from this period to the next. When the variance, η^2, is constant, (3) simplifies to $E_t\Delta c_{t+1} = \phi + \sigma R_t$, where $\phi = \frac{\eta^2}{2\sigma} - \sigma\theta$.

Campbell and Mankiw (1991) account for liquidity constraints in a simple way by assuming that aggregate consumption is equal to a weighted average of rule-of-thumb consumers and permanent income consumers with weights μ and $1-\mu$ respectively. In addition, Campbell and Mankiw (1991) assume that rule-of-thumb consumers determine consumption growth as a weighted average of income growth, current and lagged one period, with weights λ and $1-\lambda$. We can then formulate an augmented version of (3) with constant variance as

$$E_t\Delta c_{t+1} = (1-\mu)\phi + \mu[\lambda E_t\Delta y_{t+1} + (1-\lambda)\Delta y_t] + (1-\mu)\sigma R_t, \tag{4}$$

where Δy_{t+1} and Δy_t are disposable income growth at times $t+1$ and t. As stressed by Basu and Kimball (2002) and later by Galí *et al.* (2007), the interpretation of the results in Campbell and Mankiw (1991) hinges on the assumption of utility preferences that are separable in consumption and labour. Otherwise, due to the high correlation between changes in disposable income and hours worked, a significant μ may be the outcome from estimating (4) even if all consumers are fully permanent income consumers.

We may also formulate an augmented version of (4) by adding lagged change in consumption, Δc_t, and an equilibrium correction term, $(c_t - vy_t)$, such that

$$E_t\Delta c_{t+1} = (1-\mu)\phi + \mu[\lambda E_t\Delta y_{t+1} + (1-\lambda)\Delta y_t] + (1-\mu)\sigma R_t + \tau\Delta c_t + \varrho(c_t - vy_t), \tag{5}$$

where consumption and income are cointegrated with the parameter v. As pointed out by Campbell and Mankiw (1991), Δc_t would appear in (5) with $\tau > 0$ if there were important quadratic adjustment costs in consumption whereas $(c_t - vy_t)$ would appear with $\varrho < 0$ in a disequilibrium model of consumption and income.[3]

The consumption Euler equation of Smets and Wouters (2003), typically included in DSGE models, is also based on CRRA preferences. However, the utility function now also includes habit formation, hC_{t-1}, that is proportional to past consumption through the parameter h.[4] In line with Smets and Wouters (2003), we may log-linearize the Euler equation around a non-stochastic steady state such that consumption obeys $c_t = (1-\omega_1)c_{t-1} + \omega_1 E_t c_{t+1} - \omega_2 \hat{r}_t$, where $\omega_1 = (1+h)^{-1}$, $\omega_2 = \dfrac{(1-h)}{(1+h)\delta}$ and $\hat{r}_t$ is the log deviation of the *ex ante* real interest rate from its non-stochastic steady state. Consumption thus depends on a weighted average of past and expected future consumption and the *ex ante* real interest rate.

[3] Campbell and Mankiw (1991) impose $v = 1$ and find both factors to be insignificant for a number of countries. For the UK, the rejection of the equilibrium correction term is contested by Hendry (1991).

[4] We simplify matters by disregarding shocks to the subjective discount rate, which is another element entering the utility function in Smets and Wouters (2003).

The higher the degree of habit formation, the smaller the impact of the real interest rate on consumption for a given elasticity of substitution. We can also write expected consumption growth when $h \neq 0$ as

$$E_t \Delta c_{t+1} = h \Delta c_t + (1-h)\delta^{-1}\hat{r}_t. \tag{6}$$

Hence, the effects on expected consumption growth of a lagged change in consumption can either be attributed to habit formation, as in (6), or to quadratic adjustment costs in consumption, as in (5).

A nested CVAR

So far we have focused on various consumption models based on Euler equations. However, there exists a huge empirical literature initiated by Davidson *et al.* (1978) based on a theoretical framework that goes back to Keynes (1936), maintaining that current aggregate income is an important determinant of current aggregate consumption. The consumption models by Brodin and Nymoen (1992), Eitrheim *et al.* (2002), Erlandsen and Nymoen (2008) and Jansen (2013), which are all based on Norwegian data, belong to this literature. These studies have in common a Keynesian-type long-run consumption function of the form

$$c_t = \beta_y y_t + \beta_w w_t, \tag{7}$$

where c_t, y_t and w_t denote real consumption, real disposable income and real household wealth respectively. If c_t, y_t and w_t are integrated series of order one, I(1), (7) implies cointegration between the three variables with the cointegration parameters β_y and β_w for income and wealth. Both Erlandsen and Nymoen (2008) and Jansen (2013) augment (7) by the real after-tax interest rate as a separate variable to capture the possibility of long-run substitution effects in consumption. An increase in the real after-tax interest rate is assumed to make consumption today more expensive than consumption tomorrow. Hence, consumption is expected to decline. Notably, (7) and the 'saving for a rainy day' hypothesis in (2) share the same property of cointegration between consumption and income in the special case when $\beta_y = 1$ and $\beta_w = 0$.

We now formulate a general CVAR that nests all the Euler equations considered above as well as the various Keynesian-type consumption functions inherent in (7). Our point of reference is a *full* CVAR representation of a p-dimensional VAR of order k written as

$$\Delta X_t = \Pi X_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta X_{t-j} + \gamma t + \vartheta + \Phi D_t + \epsilon_t, \tag{8}$$

where Δ is the difference operator, $X_t = (c_t, y_t, w_t, R_t)'$ includes real consumption, c_t, real disposable income, y_t, real household wealth, w_t, and the real after-tax interest rate, R_t, as the modelled variables, t is a deterministic trend, Γ_j and Φ are matrices of coefficients, γ is a vector of coefficients, ϑ is a vector of intercepts, D_t is a vector of centred seasonal dummies, and ϵ_t is a vector of normally distributed random variables with expectation zero and unrestricted covariance matrix Ω. The initial observations $X_1, \ldots, X_k$ are regarded as given. The impact matrix Π has rank $0 \leqslant r \leqslant p$, and therefore can be written $\Pi = \alpha\beta'$, where α and β are $p \times r$ matrices of adjustment coefficients and cointegration coefficients, respectively, of full rank r. Drawing upon the analysis in Eitrheim *et al.* (2002), the Euler equation approach implies that consumption, wealth and the real after-tax interest rate are *not* equilibrium correcting and that income alone, in line with the 'saving for a rainy day' hypothesis in (2), *is* equilibrium correcting. These properties and the various hypotheses considered in sections 'The martingale and the 'saving for a rainy day' hypothesis' and 'Euler equations with CRRA preferences' are nested in the CVAR only when $r = 2$. By leading (8) one period and taking the conditional expectations of ΔX_{t+1}, we can write out the CVAR when $k = 2$ for notational simplicity as

$$
\begin{aligned}
E_t \begin{pmatrix} \Delta c_{t+1} \\ \Delta y_{t+1} \\ \Delta w_{t+1} \\ \Delta R_{t+1} \end{pmatrix} &= \begin{pmatrix} \alpha_{c1} & \alpha_{c2} \\ \alpha_{y1} & \alpha_{y2} \\ \alpha_{w1} & \alpha_{w2} \\ \alpha_{R1} & \alpha_{R2} \end{pmatrix} \begin{pmatrix} 1 & \beta_{y1} & 0 & \beta_{R1} \\ -1 & 1 & \beta_{w2} & \beta_{R2} \end{pmatrix} \begin{pmatrix} c_t \\ y_t \\ w_t \\ R_t \end{pmatrix} \\
&+ \begin{pmatrix} \gamma_{1,11} & \gamma_{1,12} & \gamma_{1,13} & \gamma_{1,14} \\ \gamma_{1,21} & \gamma_{1,22} & \gamma_{1,23} & \gamma_{1,24} \\ \gamma_{1,31} & \gamma_{1,32} & \gamma_{1,33} & \gamma_{1,34} \\ \gamma_{1,41} & \gamma_{1,42} & \gamma_{1,43} & \gamma_{1,44} \end{pmatrix} \begin{pmatrix} \Delta c_t \\ \Delta y_t \\ \Delta w_t \\ \Delta R_t \end{pmatrix} + \gamma t + \vartheta + \Phi D_{t+1},
\end{aligned} \tag{9}
$$

where $E_t \epsilon_{t+1} = 0$ and $\beta_{y1} = -v$ from (5). Exact identification of the two cointegrating vectors is achieved by imposing $\beta_{c1} = 1$ and $\beta_{w1} = 0$ in the first row of β' and $\beta_{c2} = -1$ and $\beta_{y2} = 1$ in the second row of β', all dictated from the theory of cointegration between consumption and income. The consumption Euler equation and the 'saving for a rainy day' hypothesis together impose $\beta_{y1} = -1$ and $\beta_{w2} = \alpha_{w1} = \alpha_{w2} = \alpha_{R1} = \alpha_{R2} = 0$ as additional restrictions on the cointegrating part of (9), which makes the two cointegrating vectors unidentifiable. Nonetheless, (9) provides important insights by deriving some of the single equation relationships in section 'Euler equations with CRRA preferences' from it. In particular, it is only when $\alpha_{c1} = \alpha_{c2}$ that consumption is *not* equilibrium correcting and this restriction can be tested empirically once the two cointegrating vectors are *exactly* identified. When $\alpha_{c1} = \alpha_{c2}$ the consumption Euler equation in the case of no rule-of-thumb consumers is given by $E_t \Delta c_{t+1} = \vartheta_c + \alpha_{c1}(\beta_{R1} + \beta_{R2})R_t$, where $\Gamma_1 = 0$, $\gamma = 0$, $\Phi = 0$, $\vartheta_c = \phi$ and $\alpha_{c1}(\beta_{R1} + \beta_{R2}) = \sigma$, in accordance with (3) with constant variance.

The 'saving for a rainy day' hypothesis is likewise given by $E_t \Delta y_{t+1} = \vartheta_y + (\alpha_{y1} - \alpha_{y2})(c_t - y_t) + (\alpha_{y1}\beta_{R1} + \alpha_{y2}\beta_{R2})R_t$, where $\vartheta_y = \varsigma$ and $(\alpha_{y1} - \alpha_{y2})^{-1} = \rho$, in line with (2). The additional term $(\alpha_{y1}\beta_{R1} + \alpha_{y2}\beta_{R2})R_t$ makes the 'saving for a rainy day' hypothesis in (9) somewhat less restrictive than (2) in the sense that the real after-tax interest rate is allowed to vary over time. The additional term is easy to handle such that the CVAR also nests all the hypotheses in (5) with some rule-of-thumb consumers. To see this, we multiply (9) by the matrix $c' = (1, -\mu\lambda, 0, 0)$, still assume $\alpha_{c1} = \alpha_{c2}$, $\beta_{y1} = -1$ and $\beta_{w2} = 0$, and rearrange terms to obtain

$$
\begin{aligned}
E_t \Delta c_{t+1} - \mu\lambda E_t \Delta y_{t+1} = \vartheta_c - \mu\lambda\vartheta_y &+ (\gamma_{1,12} - \mu\lambda\gamma_{1,22})\Delta y_t \\
&+ [\alpha_{c1}(\beta_{R1} + \beta_{R2}) - \mu\lambda(\alpha_{y1}\beta_{R1} + \alpha_{y2}\beta_{R2})]R_t \\
&+ (\gamma_{1,11} - \mu\lambda\gamma_{1,21})\Delta c_t - \mu\lambda(\alpha_{y1} - \alpha_{y2})(c_t - y_t),
\end{aligned} \tag{10}
$$

where $\gamma = 0$, $\Phi = 0$, $\vartheta_c - \mu\lambda\vartheta_y = (1-\mu)\phi$, $\gamma_{1,12} - \mu\lambda\gamma_{1,22} = \mu(1-\lambda)$, $\alpha_{c1}(\beta_{R1} + \beta_{R2}) - \mu\lambda(\alpha_{y1}\beta_{R1} + \alpha_{y2}\beta_{R2}) = (1-\mu)\sigma$, $\gamma_{1,11} - \mu\lambda\gamma_{1,21} = \tau$ and $-\mu\lambda(\alpha_{y1} - \alpha_{y2}) = \varrho$.

The theories we have discussed above imply different outcomes for subsequent empirical estimation of the consumption equation. First, a logarithmic version of the martingale hypothesis of Hall (1978), $E_t\Delta c_{t+1} = 0$, implies that $\mu\lambda$ equals zero and that no significant terms appear on the right-hand side of (9). Second, precautionary saving in response to uncertainty is reflected in the intercept, $\vartheta_c - \mu\lambda\vartheta_y$. Third, a significantly positive estimate of $[\alpha_{c1}(\beta_{R1} + \beta_{R2}) - \mu\lambda(\alpha_{y1}\beta_{R1} + \alpha_{y2}\beta_{R2})]$ can be interpreted as the intertemporal elasticity of substitution in consumption. Fourth, a significantly positive estimate of $(\gamma_{1,11} - \mu\lambda\gamma_{1,21})$ points to quadratic adjustment costs or habit formation in consumption. Fifth, significantly positive estimates of $\mu\lambda$ and $(\gamma_{1,12} - \mu\lambda\gamma_{1,22})$ indicate a substantial portion of rule-of-thumb consumers responding to current and one period lag in income growth respectively. Finally, a significantly positive estimate of $\mu\lambda(\alpha_{y1} - \alpha_{y2})$ can be interpreted as the coefficient of speed of adjustment in a disequilibrium model of consumption and income.

The Keynesian consumption function approach, as opposed to the Euler equation approach, implies that consumption *is* equilibrium correcting in the CVAR. To simplify the exposition, we now assume that $r = 1$. When the cointegration vector is normalized with respect to consumption and $k = 2$, the CVAR in (8) becomes

$$\begin{pmatrix} \Delta c_t \\ \Delta y_t \\ \Delta w_t \\ \Delta R_t \end{pmatrix} = \begin{pmatrix} \alpha_c \\ \alpha_y \\ \alpha_w \\ \alpha_R \end{pmatrix} [c_{t-1} - \beta_y y_{t-1} - \beta_w w_{t-1} - \beta_R R_{t-1}] + \Gamma_1 \begin{pmatrix} \Delta c_{t-1} \\ \Delta y_{t-1} \\ \Delta w_{t-1} \\ \Delta R_{t-1} \end{pmatrix} + \gamma t + \vartheta + \Phi D_t + \epsilon_t. \tag{11}$$

It follows that consumption is equilibrium correcting when $-1 < \alpha_c < 0$. However, income, wealth and the real after-tax interest rate may also be equilibrium correcting if the corresponding value of alpha is positive and less than one. If, on the other hand, $\alpha_y = \alpha_w = \alpha_R = 0$, then income, wealth and the real after-tax interest rate are all weakly exogenous with respect to β and the conditional Keynesian consumption function from (11) becomes

$$\begin{aligned} \Delta c_t = {} & \alpha_c[c_{t-1} - \beta_y y_{t-1} - \beta_w w_{t-1} - \beta_R R_{t-1}] + \omega_y \Delta y_t + \omega_w \Delta w_t + \omega_R \Delta R_t \\ & + \tilde{\gamma}_{1,11}\Delta c_{t-1} + \tilde{\gamma}_{1,12}\Delta y_{t-1} + \tilde{\gamma}_{1,13}\Delta w_{t-1} + \tilde{\gamma}_{1,14}\Delta R_{t-1} \\ & + \tilde{\gamma}_c t + \tilde{\vartheta}_c + \tilde{\Phi}_c D_t + \tilde{\epsilon}_{ct}, \end{aligned} \tag{12}$$

where the inclusion of the contemporaneous variables, Δy_t, Δw_t and ΔR_t, follows from the properties of the multivariate normal error distribution and where the coefficients are linear functions of the coefficients in (11) and the parameters from the multivariate normal error distribution; see for instance Johansen (1995 p. 122).

We have seen that cointegration in (8) represents the common ground between the consumption Euler equation approach and the Keynesian consumption function approach and that the theoretical predictions of the two approaches place different restrictions with respect to weak exogeneity on consumption and income. In the empirical analysis, we shall therefore consider hypotheses of cointegration and equilibrium correction as restrictions on $\Pi = \alpha\beta'$ in order to discriminate between the two approaches. Because CVAR models considering conditional expectations of future consumption and income may corroborate parameters stemming from the class of Euler equations, we shall also examine the empirical relevance of such models within the context of Johansen and Swensen (1999, 2008).

III. Data

For reasons of comparison, we maintain the data set of Jansen (2013) *as is* and extend it by using quarterly growth rates from the final national accounts for the period $2008q3 - 2016q4$, keeping $2008q2$ fixed. Because the capital markets in Norway were heavily regulated during the 1970s and early 1980s, which likely prevented many consumers from acting freely in accordance with a consumption Euler equation, we choose $1984q1$ as the starting point of our sample period. However, due to lags in the CVAR, the sample period for estimation purposes includes data points from $1982q3$ to $2016q4$. The sample period is thus consistent with the period of liberalization of what were believed to be the most binding regulations of credit for households, namely the bond market, which was deregulated in several steps between 1982 and 1985 to allow for competition among banks and other lending institutions in the household market. We also choose $2008q4$ as the starting point of the financial crisis. Although the bankruptcy of Lehman Brothers took place on 15 September 2008, we believe that the main effects on the Norwegian economy, and hence on household consumer behaviour, were felt from the fourth quarter of 2008 onwards.

The consumption variable is defined as real consumption excluding expenditure on health services and housing. The income variable is real disposable income excluding equity income. The wealth variable is measured in real terms net of household debt and thus consists of the value of housing plus total net financial wealth. Finally, the real after-tax interest rate is defined as the average nominal interest rates on bank loans faced by households net of marginal income tax and adjusted for inflation. In Online Appendix A, we give more precise definitions of all the variables in the empirical models in sections IV and V.

Figure 1 shows the consumption to income and the wealth to income ratios together with the real after-tax interest rate for the sample period $1982q3-2016q4$. We observe a strong co-movement between the two ratios in the sample period before the financial crisis hit the Norwegian economy and this is *prima facie* evidence of cointegration between the three variables involved. However, a break in the cointegration relationship seems evident in the subsequent period as the two ratios then diverge and move in opposite directions. The real after-tax interest rate, for its part, reached a historically high level in the early 1990s in the wake of the huge boom in consumption during the second half of the 1980s. Since then the real after-tax interest rate has shown a downward trend and reached negative levels, as it did in the early 1980s, at the end of the sample period. These features of the data are the premises for the cointegration analysis and the modelling of short-run dynamics.

IV. Cointegration and dynamics[5]

In this section, we first carry out a multivariate cointegration analysis with a structural break around the time of the financial crisis in 2008, applying the models and methods in

[5]The econometric modelling in this section was carried out with PcGive 14; see Doornik and Hendry (2013).

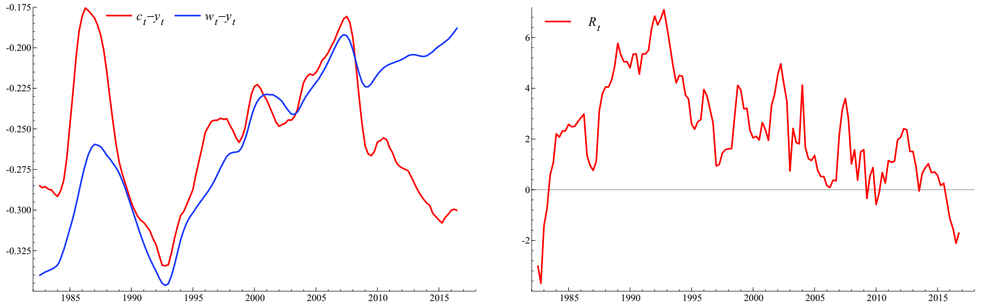

Figure 1. Consumption to income ($c_t - y_t$) ratio, wealth to income ($w_t - y_t$) ratio and real after-tax interest rate (R_t)
Notes: Sample period: 1982q3−2016q4. The left panel shows the moving averages of the two ratios as logarithms, with one quarter lag and two quarters lead. The mean and range of the logarithms of wealth to income are matched to the mean and range of the logarithms of consumption to income. The right panel shows the real after-tax interest rate measured in per cent per annum.

Johansen *et al.* (2000). Then we estimate consumption short-run dynamics within a partial CVAR, following the modelling strategy in Harbo *et al.* (1998), to calculate the magnitude of the MPC.

Test results

A preliminary analysis of $\Pi = \alpha\beta'$, using (8) as the underlying model with $k = 6$ guided by Akaike's information criterion (AIC), likelihood ratio tests of sequential model reduction and diagnostic tests of the residuals, confirms a significant structural break in the long-run relationship around the time of the financial crisis.[6] We are therefore motivated to follow Johansen *et al.* (2000) and capture a structural break in the long-run relationship by means of a model which takes into account the possibility of separate trends in the two periods $1, \ldots, T_1$ and $T_1 + 1, \ldots, T$. The idea is to allow for two VAR models where the k first observations are conditioned upon, but where the parameters of the stochastic components are the same for both models, and where the parameters of the deterministic components may differ, corresponding to a broken trend. Formally, let $T_0 = 0$ and $T_2 = T$. If $ID_{j,t} = 1$ for $t = T_{j-1}$ and $ID_{j,t} = 0$ otherwise so that $ID_{j,t-i}$ is the indicator for the ith observation in the jth period, $j = 1, 2$, it follows that $SD_{j,t} = \Sigma_{i=k+1}^{T_j - T_{j-1}} ID_{j,t-i} = 1$ for $t = T_{j-1} + k + 1, \ldots, T_j$ and $SD_{j,t} = 0$ otherwise. The CVAR in (8) is then reformulated for $t = k + 1, \ldots, T$ as

$$\Delta X_t = \alpha \begin{pmatrix} \beta \\ \gamma \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ tSD_t \end{pmatrix} + \mu SD_t + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-(k-1)} + \Phi D_t + \kappa_{2,1} ID_{2,t-1} + \ldots + \kappa_{2,k} ID_{2,t-k} + \epsilon_t, \tag{13}$$

where $SD_t = (SD_{1,t}, SD_{2,t})'$, $\gamma = (\gamma_1', \gamma_2')'$ and $\mu = (\mu_1, \mu_2)$. We assume, referring to Figure 1 and the discussion in section III, that the break occurs in 2008q4. Accordingly, $SD_{1,t}$ is a step dummy which equals one in the period 1982q3−2008q3, $SD_{2,t}$ is a step dummy which

[6] Detailed results can be found in Online Appendix B.

TABLE 1

Trace test results for cointegration with a structural break[1]

Eigenvalue (λ_i)	H_0	H_A	λ_{trace}
0.287	$r=0$	$r\geqslant 1$	101.21 [0.001]
0.188	$r\leqslant 1$	$r\geqslant 2$	56.49 [0.050]
0.129	$r\leqslant 2$	$r\geqslant 3$	28.96 [0.207]
0.078	$r\leqslant 3$	$r=4$	10.74 [0.372]

Diagnostics[2]	*Test statistic*	*p-value*
Vector autocorrelation 1-5 test:	F(80,286)=1.37	0.034
Vector normality test:	$\chi^2(8)=9.02$	0.341
Vector heteroscedasticity test:	F(224,266)=1.03	0.407

Notes: Sample period: 1982q3$-$2016q4. [1] See Johansen *et al.* (2000), VAR of order 6, modelled variables: c_t, y_t, w_t and R_t, deterministic variables: tSD_t (restricted), SD_t (unrestricted), $ID_{2,t}$ (unrestricted) and centred seasonal dummies (unrestricted), r denotes the rank order of $\Pi=\alpha\beta'$ and λ_{trace} is the trace statistic with p-value in square brackets, which are calculated by means of the estimated response surface in Johansen *et al.* (2000 Table 4). [2] See Doornik and Hendry (2013 p. 172).

equals one in the period 2010q2$-$2016q4 and $ID_{2,t}$ are impulse indicators which equal one for $t=2008q4,...,2010q1$, otherwise zero for $k=6$.

We can now conduct a cointegration analysis with the augmented VAR, letting SD_t and $ID_{2,t}$ enter unrestrictedly, whereas tSD_t is restricted to lying within the cointegration space. Again, according to both the AIC and the series of model reduction tests, the VAR in our case should include six lags as the premise for the cointegration analysis. Moreover, with $k=6$ there are no serious departures from white noise residuals according to the diagnostic tests. Juselius (2006 p. 72) suggests as a rule of thumb using a VAR with $k=2$ in a tentative cointegration analysis. We find, however, that such a specification suffers from severe residual autocorrelation. Because (13) controls for a structural break around the time of the financial crisis and both the fourth and the fifth lag of consumption dynamics are strongly significant in the estimated models in sections 'Short run dynamics' and 'Estimated CVAR models', we argue that the severe residual autocorrelation is associated with omitted dynamics rather than structural misspecification. We therefore maintain that cointegration results based on VAR(6) are more likely to represent the underlying data structure.[7]

Table 1 shows trace test results for cointegration together with the diagnostic tests of the underlying VAR. The trace tests marginally support the hypothesis of two cointegrating vectors between c_t, y_t, w_t and R_t at the 5% significance level. We shall therefore consider two cases, $r=2$ and $r=1$, when testing restrictions on $\Pi=\alpha\beta'$ to discriminate between the consumption Euler equation and the Keynesian consumption function. Starting with $r=2$, we find, once the two cointegrating vectors are exactly identified as described below (9), that the restriction $\alpha_{c1}=\alpha_{c2}$ is strongly rejected by a likelihood ratio statistic, which becomes $\chi^2(1)=10.65$ with a p-value of 0.001. We can conclude already at this stage of the analysis that the data overwhelmingly reject a consumption Euler equation.

[7] A comparative cointegration analysis with VAR(2) together with a summary of the AIC, the model reduction tests and diagnostic tests are given in Online Appendix C.

TABLE 2

Likelihood ratio test results for restrictions on $\Pi = \alpha\beta'$ with a structural break[1]

Model (i): $\beta_c = 1$

$$c_{t-1} - \underset{(0.23)}{1.06} y_{t-1} - \underset{(0.039)}{0.16} w_{t-1} + \underset{(0.32)}{1.72} R_{t-1} + \underset{(0.0013)}{0.0023} tSD_{1,t} + \underset{(0.0020)}{0.0066} tSD_{2,t}$$

$$\hat{\alpha}_c = -\underset{(0.09)}{0.31},\ \hat{\alpha}_y = \underset{(0.07)}{0.003},\ \hat{\alpha}_w = -\underset{(0.11)}{0.27},\ \hat{\alpha}_R = -\underset{(0.03)}{0.15}$$

$\log L = 1558.35$

Model (ii): $\beta_c = 1$, $\beta_y + \beta_w = 1$

$$c_{t-1} - 0.84 y_{t-1} - \underset{(0.04)}{0.16} w_{t-1} + \underset{(0.35)}{1.97} R_{t-1} + \underset{(0.00028)}{0.00089} tSD_{1,t} + \underset{(0.001)}{0.0051} tSD_{2,t}$$

$$\hat{\alpha}_c = -\underset{(0.08)}{0.26},\ \hat{\alpha}_y = \underset{(0.07)}{-0.008},\ \hat{\alpha}_w = -\underset{(0.10)}{0.24},\ \hat{\alpha}_R = -\underset{(0.03)}{0.14}$$

$\log L = 1557.71$

$\chi^2(1) = 1.28[0.26]$[2]

Model (iii): $\beta_c = 1$, $\beta_y = 1$, $\beta_w = 0$

$$c_{t-1} - y_{t-1} + \underset{(0.63)}{4.28} R_{t-1} + \underset{(0.00035)}{0.00026} tSD_{1,t} + \underset{(0.0021)}{0.0074} tSD_{2,t}$$

$$\hat{\alpha}_c = -\underset{(0.04)}{0.10},\ \hat{\alpha}_y = \underset{(0.03)}{-0.026},\ \hat{\alpha}_w = -\underset{(0.05)}{0.15},\ \hat{\alpha}_R = -\underset{(0.01)}{0.07}$$

$\log L = 1556.74$

$\chi^2(2) = 3.21[0.20]$[3], $\chi^2(1) = 2.16[0.14]$[4]

Model (iv): $\beta_c = 1$, $\beta_y + \beta_w = 1$, $\alpha_y = 0$

$$c_{t-1} - 0.83 y_{t-1} - \underset{(0.04)}{0.17} w_{t-1} + \underset{(0.35)}{1.93} R_{t-1} + \underset{(0.00028)}{0.00091} tSD_{1,t} + \underset{(0.00099)}{0.0050} tSD_{2,t}$$

$$\hat{\alpha}_c = -\underset{(0.08)}{0.26},\ \hat{\alpha}_w = -\underset{(0.10)}{0.24},\ \hat{\alpha}_R = -\underset{(0.03)}{0.15}$$

$\log L = 1557.70$

$\chi^2(2) = 1.28[0.53]$[5], $\chi^2(1) = 0.002[0.97]$[6]

Notes: Sample period: $1982q3-2016q4$. [1] See Johansen *et al.* (2000), VAR of order 6 with a structural break in $2008q4$, $r = 1$, modelled variables: c_t, y_t, w_t and R_t, deterministic variables: $tSD_{1,t}$ and $tSD_{2,t}$ (restricted), $SD_{1,t}$ and $SD_{2,t}$ (unrestricted), $ID_{2,t}$ (unrestricted) and centred seasonal dummies (unrestricted), standard errors in parentheses, p-values in square brackets. [2] $\beta_y + \beta_w = 1$. [3] $\beta_y = 1$, $\beta_w = 0$. [4] $\beta_w = 0$. [5] $\beta_y + \beta_w = 1$, $\alpha_y = 0$. [6] $\alpha_y = 0$.

Table 2 reports main likelihood ratio tests of restrictions on $\Pi = \alpha\beta'$, assuming $r = 1$. The hypothesis of homogeneity between consumption, income and wealth is not rejected by the data in Model (ii). Note that the trend variable for the first period is *not* excluded from the model, as the estimate of γ_1 is a borderline case at the 10% significance level (p-value = 0.103). A likelihood ratio test, $\chi^2(1) = 1.94$ and p-value = 0.16, supports reduction from Model (ii) to Model (iii), in which homogeneity between consumption and income and exclusion of the wealth variable are imposed. However, the p-value of the individual hypothesis $\beta_w = 0$ equals 0.14 and the associated t-value is as high as 4 in Model (ii). In addition, the estimates of β_R and α change significantly when homogeneity is imposed only between consumption and income. We therefore keep the wealth variable in the cointegrating vector and find that the estimated adjustment coefficients, apart from the estimate of α_y (p-value = 0.97), are all highly significant in Model (iv). Accordingly, consumption, and not income, equilibrium corrects in the CVAR, which clearly contradicts the consumption Euler equation. When homogeneity is imposed between consumption, income and wealth together with weak exogeneity of income, the restricted long-run relationship becomes

$$\widehat{eqcm}_t = c_{t-1} - 0.83y_{t-1} - 0.17w_{t-1} + 1.93R_{t-1} + 0.00091tSD_{1,t} + 0.0050tSD_{2,t}. \quad (14)$$

Recursively estimated coefficients of y_t, and hence also of w_t, as well as of R_t, are stable before, during and after the financial crisis once the structural break is allowed for. Also, recursive likelihood ratio tests support the joint hypothesis of $\beta_y + \beta_w = 1$ and $\alpha_y = 0$.[8] A comparison with equation (2) in Jansen (2013) shows that the estimated long-run coefficients of income and wealth are almost perfectly reproduced for the sample period ending in 2016q4. We also find that the deterministic trend in (14) significantly shifts equilibrium consumption downwards both before and after the financial crisis. However, the shift is much larger after 2008q4, with a factor of 5.5 according to model (*iv*). A possible interpretation may be that the broken trend reflects increased uncertainty and thus increased precautionary saving in the wake of the financial crisis. The fact that the household saving ratio increased from nearly 4% in 2008 to more than 10% in 2015 supports this conjecture.

Short-run dynamics

To facilitate a comparison of the magnitude of the MPC implied by Model A1 in table 4 in Jansen (2013), we perform a reduced rank regression for a partial model, following the modelling strategy of equation (10) in Harbo *et al.* (1998). Since the hypothesis of weak exogeneity of income with respect to the long-run parameters is supported by the data, we can condition on this variable without loss of information when estimating a partial CVAR for consumption, wealth and the real after-tax interest rate. Our point of departure is therefore the partial model written in vector form as

$$\Delta X_t^* = \Theta_D D_t + \sum_{j=0}^{5} \omega_{Z,j} \Delta Z_{t-j} + \sum_{j=1}^{5} \Theta_{X^*,j} \Delta X_{t-j}^* + \alpha \begin{pmatrix} \beta \\ \gamma \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ tSD_t \end{pmatrix} + \varepsilon_t, \quad (15)$$

where $X_t^* = (c_t, w_t, R_t)'$, $Z_t = y_t$, $X_t = (c_t, y_t, w_t, R_t)'$ and D_t includes the centred seasonal dummies and all the dummies for the structural break around the time of the financial crisis. First, we estimate (15) by means of constrained full information maximum likelihood (CFIML) whereby the rank is restricted to one and the hypothesis of homogeneity between consumption, income and wealth is imposed in accordance with the evidence above. Then, we simplify the model, general-to-specific, by deleting the most *in*significant short-run dynamics one-by-one within the system as a whole. The simplified dynamic consumption model with respect to the stochastic variables and the broken trend in the long-run relationship becomes

$$\begin{aligned} \Delta\hat{c}_t = & -\underset{(0.09)}{0.34}\,\Delta c_{t-1} - \underset{(0.07)}{0.14}\,\Delta c_{t-2} + \underset{(0.07)}{0.45}\,\Delta c_{t-4} + \underset{(0.08)}{0.25}\,\Delta c_{t-5} + \underset{(0.09)}{0.21}\,\Delta y_t \\ & + \underset{(0.10)}{0.27}\,\Delta y_{t-3} + \underset{(0.11)}{0.24}\,\Delta y_{t-4} + \underset{(0.07)}{0.26}\,\Delta w_{t-1} - \underset{(0.07)}{0.15}\,\Delta w_{t-4} - \underset{(0.10)}{0.34}\,c_{t-1} \\ & + \underset{(0.07)}{0.26}\,y_{t-1} + \underset{(0.03)}{0.08}\,w_{t-1} - \underset{(0.11)}{0.28}\,R_{t-1} - \underset{(0.0001)}{0.0004}\,tSD_{1,t} - \underset{(0.0005)}{0.0016}\,tSD_{2,t}. \end{aligned} \quad (16)$$

[8] See Online Appendix D.

The consumption model seems quite well-specified, judging by diagnostics tests and plots of model fit and residuals.[9] Interestingly, (16) implies a first year MPC of around 40%, which is quite close to the 30% implied by Model A1 in Jansen (2013) and recent micro-econometric evidence referred to in section I. These findings are in line with the argument in Doornik and Hendry (1997) that the main source of forecast failure is deterministic shifts in equilibrium means, for example the equilibrium saving ratio, and not shifts in the derivative coefficients, for example the marginal propensity to consume, that are of primary interest for policy analysis.

So far we have modelled the financial crisis as a structural break in the long-run relationship between consumption, income, wealth and the real interest rate. As a final exercise, we test for parameter constancy of the short-run dynamic wealth effects, which one could argue have changed after the financial crisis. Indeed, adding lagged effects of the interaction between $SD_{2,t}$ and Δw_t in the whole specific system, the fourth lag emerged with a significant and positively signed parameter estimate in the consumption model only.[10] A possible interpretation of the fact that the short-run response of consumption to wealth has increased since the financial crisis is the following: Since the financial crisis, households have faced increased credit constraints in the form of *inter alia* lending criteria based on payment-to-income ratios due to increased credit risk in the economy. Households have thus not been able to borrow at the observed lending interest rates as easily as before the financial crisis because of tightening of credit practices. As a consequence, household credit worthiness, as measured by total wealth, has become increasingly important for mortgage and other loan security since the financial crisis, and thus also for the ability to borrow for consumption purposes in the short run.

Before turning to conditional expectations of future consumption and income, one may also interpret the rejection of a consumption Euler equation as being due to increased credit constraints since the financial crisis, which has made it much more difficult for households to smooth consumption. However, redoing our cointegration analysis with a sample period ending in 2008q3, as shown in Online Appendix B, leads to the same conclusions regarding the validity of the consumption Euler equation. For example, the hypothesis $\alpha_{c1} = \alpha_{c2}$ is even more strongly rejected for this shorter sample period than for the whole sample. Our results are thus quite robust to the choice of sample period.

V. Conditional expectations[11]

Although the findings from testing restrictions on $\Pi = \alpha\beta'$ do not support a consumption Euler equation, the question of whether conditional expectations of future consumption and income play a role in explaining consumer behaviour is left unanswered. We recall from section 'A nested CVAR' that the conditional expectations of future consumption and income in (10) nests all the hypotheses in (5) with some rule-of-thumb consumers.

[9] Detailed estimation results, diagnostic tests and plots of model fit and residuals of the whole specific system are reported in Online Appendix E.

[10] Detailed results are given in Online Appendix F.

[11] The estimation and testing in this section are performed with the statistical package R, see R Core Team (2019).

Conditional expectations of future consumption and income can be treated within the context of Johansen and Swensen (1999, 2008). First, we outline the estimation and testing procedure, paying particular attention to the conditional expectations restrictions on the stochastic part of the CVAR. Then, we estimate CVAR models using (5) with conditional expectations as the reference point and examine whether data can corroborate parameters stemming from the class of Euler equations. Throughout the analysis, we specify the *exact* form of CVAR models in which the rank order of the impact matrix is one and income is not weakly exogenous as in section ‘Short run dynamics’.

Outline of the estimation and testing procedure

To simplify the exposition, we outline the estimation and testing procedure by means of (8). The consumption Euler equations involving expectations of future variables can generally be expressed as $c'E_t\Delta X_{t+1}=d'X_t$, which implies restrictions on the coefficients in (8). For instance, a bivariate system in which the variables satisfy a martingale hypothesis can be written $E_tX_{1,t+1}=(1,0)E_t(X_{1,t+1},X_{2,t+1})'=(1,0)(X_{1,t},X_{2,t})'=X_{1,t}$ or $(1,0)E_t\Delta(X_{1,t+1},X_{2,t+1})'=0$. It is often convenient to have a more general specification of the form

$$c'E_t\Delta X_{t+1}-d'X_t+d'_{-1}\Delta X_t+\ldots+d'_{-k+1}\Delta X_{t-k+2}+\vartheta_0+\gamma_0 t+\Phi_0 D_t=0 \qquad (17)$$

where c, d, $d_{-1},\ldots,d_{-k+1}$, ϑ_0, γ_0 and Φ_0 have known elements.

A flexible formulation is achieved by assuming that the $p\times q$ matrix c is known and allowing d, $d_{-1},\ldots,d_{-k+1}$, ϑ_0, γ_0 and Φ_0 to be treated as matrices of unknown parameters. If they are allowed to vary freely, (17) does not imply any constraints. By testing whether any of the matrices d, $d_{-1},\ldots,d_{-k+1}$, ϑ_0, γ_0 and Φ_0 in (17) are equal to zero, or any given matrix, one can investigate whether a simplification of the conditional expectations is possible.

Using the methods described in Johansen and Swensen (1999, 2008), the value of the concentrated likelihood $L_c(d,d_{-1},\ldots,d_{-k+1},\vartheta_0,\gamma_0,\Phi_0)$ can be computed. Further maximization over $d,d_{-1},\ldots,d_{-k+1},\vartheta_0,\gamma_0$ and Φ_0 yields a value $\max L_c(d,d_{-1},\ldots,d_{-k+1},\vartheta_0,\gamma_0,\Phi_0)$, which is equal to the maximum value of the likelihood for (8), denoted L_{max}. The likelihood ratio for a test of a particular hypothesis, for instance $d_{-k+1}=d^0_{-k+1}$, can be found as

$$\frac{\max_{d,d_{-1},\ldots,d_{-k+2},\vartheta_0,\gamma_0,\Phi_0} L_c(d,d_{-1},\ldots,d^0_{-k+1},\vartheta_0,\gamma_0,\Phi_0)}{\max_{d,d_{-1},\ldots,d_{-k+2},d_{-k+1},\vartheta_0,\gamma_0,\Phi_0} L_c(d,d_{-1},\ldots,d_{-k+1},\vartheta_0,\gamma_0,\Phi_0)} =\frac{\max_{d,d_{-1},\ldots,d_{-k+2},\vartheta_0,\gamma_0,\Phi_0} L_c(d,d_{-1},\ldots,d^0_{-k+1},\vartheta_0,\gamma_0,\Phi_0)}{L_{max}}.$$

The maximization with respect to $d_{-1},\ldots,d_{-k+2},\vartheta_0,\gamma_0$ and Φ_0 can be performed by means of ordinary least squares (OLS) and reduced rank regression, while maximizing with respect to d must be carried out using numerical optimization. A more detailed explanation of the procedure can be found in Online Appendix G.

TABLE 3

Likelihood ratio test results for simplifying restrictions[1]

Model	*Restrictions*	$logL_i$	$i-j$[2]	$-2\log\frac{L_j}{L_i}$	*df*	*p-value*
1	–	1558.35	–	–	–	–
2	$\beta_y+\beta_w=1$	1557.71	1–2	1.28	1	0.26
3	Model 2, $\gamma_{5,14}=0$	1557.69	2–3	0.04	1	0.84
4	Model 3, $\gamma_{5,13}=0$	1556.55	3–4	2.28	1	0.13
5	Model 4, $\gamma_{5,12}=0$	1554.87	4–5	3.36	1	0.07
6	Model 5, $\gamma_{5,11}=0$	1549.34	5–6	11.06	1	0.0009
7	Model 5, $\gamma_{4,14}=0$	1554.08	5–7	1.58	1	0.21
8	Model 7, $\gamma_{4,13}=0$	1550.13	7–8	7.90	1	0.005
9	Model 7, $\gamma_{3,14}=0$	1554.06	7–9	0.04	1	0.84
10	Model 9, $\gamma_{2,14}=0$	1552.99	9–10	2.14	1	0.14
11	Model 10, $\gamma_{1,14}=0$	1551.06	10–11	3.86	1	0.05

Notes: Sample period: $1982q3-2016q4$. [1] See Johansen and Swensen (1999, 2008). [2] $i-j$ denotes the likelihood ratio test for the additional restriction(s) on model j compared to model i.

Estimated CVAR models

The conditional expectations, $c'E_t\Delta X_{t+1}$, can also be found from (13) by leading the variables one period and taking expectations at time t. Comparing the coefficients from an augmented version of (17), where a broken trend is taken into account, implies the following restrictions on the stochastic variables

$$c'\alpha\beta'=d', c'\Gamma_1=-d'_{-1},\ldots,c'\Gamma_{k-1}=-d'_{-k+1}, \tag{18}$$

while the non-stochastic variables are unspecified and have the form $c'\alpha\gamma'(tSD_{t+1}+SD_{t+1})+c'\Phi D_{t+1}+c'\mu SD_{t+1}+c'\kappa_{2,1}ID_{2,t}+\ldots+c'\kappa_{2,k}ID_{2,t-k+1}$. As indicated in the introduction, it is reasonable to start by considering the coefficients of the stochastic variables as the parameters of interest.

We first investigate the case of conditional expectations involving only consumption, $c=(1,0,0,0)'$, and focus on the consumption equation, not the whole system as in section 'Short run dynamics', when deleting any insignificant short-run dynamics from the model, that is any insignificant coefficients in the first row of the matrices Γ_j denoted $(\gamma_{j,11},\gamma_{j,12},\gamma_{j,13},\gamma_{j,14})$ for $j=1,\ldots,5$. Specifically, we use a sequential reduction procedure, starting with the coefficients of the last lag of change in the real interest rate, that is $\gamma_{5,14}$.

Table 3 shows likelihood ratio test results for simplifying restrictions on $c'\Gamma_1=-d'_{-1},\ldots,c'\Gamma_{k-1}=-d_{-k+1}$. As pointed out earlier, when maximizing first over $d'_{-1},\ldots,d'_{-k+1}$ and the coefficients of the non-stochastic variables and then over the parameters of the cointegration vector, the value of the maximum of the likelihood is the same as for (13), see Table 2 models (*i*) and (*ii*). By sequentially simplifying the model, we end up with Model 11 which includes five lags of consumption growth, four lags of income and wealth growth and no lag of change in the real interest rate. To further simplify the remaining short-run dynamics of Model 11, all coefficients with a p-value larger than 0.1 are set equal to zero. For the stochastic variables and the broken trend in the long-run relationship, the result is

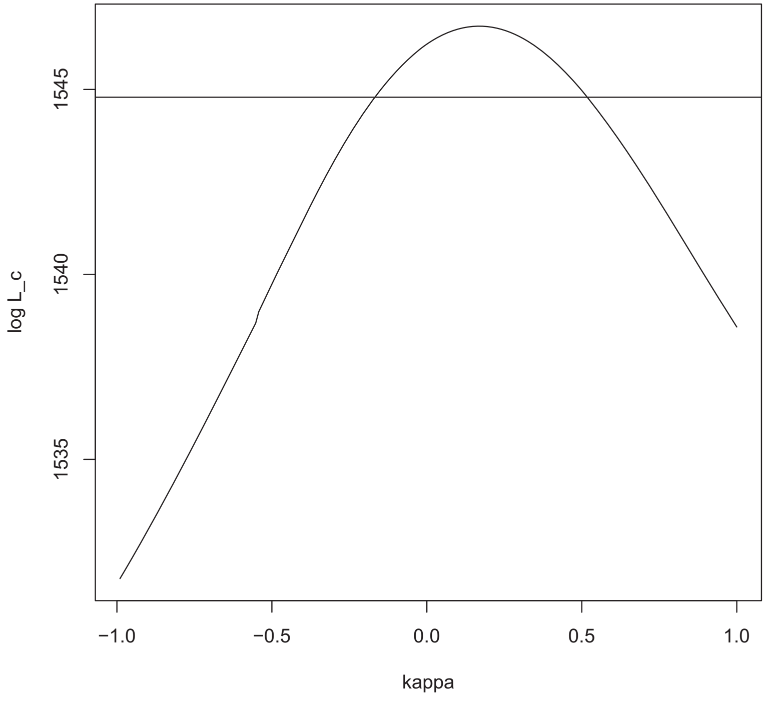

Figure 2. Concentrated log likelihood for $\kappa = \mu\lambda$ with a 90% confidence interval
Notes: Sample period: 1982q3−2016q4.

$$\begin{aligned}\widehat{E_t\Delta c_{t+1}} = & -0.16(1.0, -0.83, -0.17, 2.76, 0.0010, 0.0059)\begin{pmatrix} X_t \\ tSD_{t+1}\end{pmatrix} \\ & -\underset{(0.08)}{0.45}\,\Delta c_t - \underset{(0.07)}{0.27}\,\Delta c_{t-1} + \underset{(0.07)}{0.53}\,\Delta c_{t-3} + \underset{(0.08)}{0.30}\,\Delta c_{t-4} \\ & + \underset{(0.09)}{0.14}\,\Delta y_{t-3} + \underset{(0.07)}{0.24}\,\Delta w_t - \underset{(0.07)}{0.20}\,\Delta w_{t-3},\end{aligned} \tag{19}$$

where $X_t = (c_t, y_t, w_t, R_t)'$ and $tSD_{t+1} = (tSD_{1,t+1}, tSD_{2,t+1})'$. The corresponding maximum value of the log likelihood is 1546.23. The further reduction of Model 11 is thus valid by a likelihood ratio test with $\chi^2(6) = 9.66$ and $p = 0.14$. There are several interesting consequences of (19). The first is a clear rejection of the hypothesis that log consumption is a martingale, $E_t\Delta c_{t+1} = 0$, which is a variant of the hypothesis of Hall (1978) in (1). The result found in Jansen (2013) is therefore confirmed for the whole period. The second consequence is a clear rejection of the implicit restrictions of the consumption Euler equation in (3) with constant variance since (19) contains significant coefficients involving growth in both income and wealth. The third is that the significant coefficients of lagged consumption growth can be interpreted as habit formation in line with the Smets and Wouters (2003) model in (6).

We next investigate the case of conditional expectations involving both future consumption and income, $c = (1, -\mu\lambda, 0, 0)'$, to shed light on the magnitude of the proportion of rule-of-thumb consumers. Accordingly, we analyse a simplified version of (5) involving only $\kappa = \mu\lambda$ and not all the economically interesting parameters $\mu, \phi, \lambda, \sigma, \tau$ and ϱ separately. Figure 2 plots the concentrated log likelihood for $\kappa = \mu\lambda$ in the region

$[-1, 1]$ with $c=(1, -\mu\lambda, 0, 0)'$, using the same variables as in (19). The maximum value of the log likelihood is 1546.71, corresponding to the maximum likelihood estimate $\hat{\kappa}=0.17$. The indicated 90% confidence interval $(-0.11, 0.46)$ contains parameters which have a reasonable economic interpretation. However, since the interval also contains $\kappa=0$ this parameter cannot be claimed to be different from zero. More precisely, the likelihood ratio statistic $2(1546.71-1546.23)=0.96$ corresponds to a p-value of 0.33. The conclusions drawn from (13) with $c=(1, 0, 0, 0)'$ concerning the martingale hypothesis, the consumption Euler equation in (3) and habit formation are therefore still valid.

Treating the coefficients of the non-stochastic variables as parameters of interest also yields some conclusions worth mentioning. The step dummies SD_t are introduced to take the financial crisis into account. It is a generally accepted fact that the financial crisis was unanticipated. Therefore, the coefficients of SD_{t+1} should satisfy $c'\alpha\gamma_1=c'\alpha\gamma_2$ and $c'\mu_1=c'\mu_2$, that is that there is only a linear trend, such that the time of the break cannot be known. However, this hypothesis is overwhelmingly rejected, with a likelihood ratio statistic $2(1546.23-1540.55)=11.36$ and a corresponding p-value of 0.003 with 2 degrees of freedom. The rejection of a hypothesis that is true, *i.e.* that the financial crisis was unanticipated, provides additional support for the conclusion in section IV that a consumpton Euler equation is not a valid empirical description of the data.

We may also ask if the habit formation type behaviour in (19) is a consequence of assuming that the agents know the financial shift occurred in the way expectations are formulated in the model. However, estimating the specific consumption model using a sample period ending in 2008q3, we find that the lag structure of consumption growth is fairly similar to that in (19) for the whole sample period.[12] These findings indicate that the estimated habit formation type behaviour is not due to the way expectations about the financial shift are formulated in the model.

We conclude from all the findings in this section that most of the parameters stemming from the class of consumption Euler equations are not supported by the data when conditional expectations of consumption and income in CVAR models are considered. Only habit formation seems to play an important role in explaining Norwegian consumer behaviour.

VI. Conclusions

In this paper, we have formulated a general CVAR that nests both a class of consumption Euler equations and various Keynesian-type consumption functions. Using likelihood-based methods and Norwegian data, we found evidence of cointegration between consumption, income and wealth once a structural break around the time of the financial crisis is accounted for. The fact that consumption cointegrates with both income and wealth, and not only with income, demonstrates the empirical irrelevance of a consumption Euler equation. More importantly, we found that consumption equilibrium corrects to changes in income and wealth, but not that income equilibrium corrects to changes in consumption, as would

[12] Detailed results are given in Online Appendix H.

be the case when an Euler equation is true. Finally, we found that most of the parameters stemming from the class of Euler equations are not corroborated by the data when conditional expectations of future consumption and income in CVAR models are considered. Only habit formation, typically included in DSGE models, seems to be important in explaining Norwegian consumer behaviour. Our estimated conditional Keynesian-type consumption function implies a first year MPC of around 40%.

We have relied on a CVAR in which a structural break in the cointegration relationship between consumption, income and wealth around the event of the financial crisis has been accounted for by a broken trend. A possible interpretation may be that the broken trend reflects increased uncertainty and thus increased precautionary saving in the wake of the financial crisis. Another possibility is that the broken trend picks up some important effects of omitted variables that are necessary to explain the changed consumer behaviour since the financial crisis. For instance, we have neither included a variable capturing the changing credit conditions faced by households nor disaggregated the wealth variable into separate variables for liquid assets, illiquid assets, debt and housing. Such variables may be important in a CVAR to adequately capture the effects of the household financial accelerator on consumption in the wake of the financial crisis. We leave this issue for future work.

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Supporting Information

Additional supporting information may be found in the online version of this article:

Appendix A: Data definitions and sources.
Appendix B: A preliminary cointegration analysis.
Appendix C: A comparative cointegration analysis.
Appendix D: Recursive estimation and test statistics.
Appendix E: Detailed empirical results.
Appendix F: Analysis of parameter constancy of short-run dynamics.
Appendix G: Details of the estimation and testing procedure.
Appendix H: Additional results on conditional expectations.