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Marginal compensated effects in discrete labor supply models

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ABSTRACT

This paper develops analytic results for marginal compensated effects in discrete labor supply models, including a Slutsky equation. The Slutsky equation is aggregate in the sense that it establishes the relationship between the marginal compensated effects of the probability of working and the mean hours of work in terms of the corresponding marginal uncompensated effects. The Slutsky equation differs somewhat from the Slutsky equation in the standard continuous labor supply models. Specifically, the marginal compensated effect of an increase in the wage rate differs from the corresponding effect of a decrease in the wage rate.

To illustrate some qualitative properties of the compensated marginal effects we have used an empirical labor supply model to compute numerical compensated (Hicksian) and uncompensated marginal (Marshallian) effects resulting from wage rate changes.

1. Introduction

In microeconomic theory the Slutsky equation plays a fundamental role in the task of calculating compensated effects (Slutsky, 1915; Hicks, 1936; and Varian, 1992). It allows the researcher to compute marginal compensated (Hicksian) price and wage effects (which are unobservable) by utilizing the corresponding Marshallian marginal effects. In labor market analysis the Slutsky equation allows the researcher to break down total effects of changes in the budget constraint in income and substitution effects. The compensated labor supply elasticity is one of the key parameters in the calculation of the deadweight loss of taxation and in devising an optimal income tax policy (Feldstein, 1999, and Saez, 2001), and in the calculation of the marginal cost of public funds (Harberger, 1964; Browning, 1976; Mayshar, 1990; Jacobs, 2018). In several countries, reforms have been designed to improve work incentives. One type of these "make-work-pay" instruments is so-called in-work benefits. Unlike the traditional social transfers that are commonly paid to those out of work, the key distinguishing feature of in-work benefits is that they are conditional on working. Using the Slutsky equation, one can evaluate the substitution and income effect from this, and similar types of reforms.

In standard labor supply models, where the model relation that determines hours of work often is given by a closed-form expression, it is straightforward to calculate compensated marginal effects. In discrete labor supply models, which are based on the theory of discrete choice (McFadden, 2001), calculation of compensating effects is more complicated. Here, the labor supply function is a non-differentiable stochastic function that cannot be expressed on closed form. However, one can compute marginal compensated effects of aggregate labor supply, such as the marginal compensated effect of the probability of working and of mean hours of work.

In this paper we have used results obtained by Dagsvik and Karlström (2005) to derive compensated wage elasticities for discrete

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labor supply models and the aggregate Slutsky equations for these kinds of models. The compensated wage elasticities as well as the aggregate Slutsky equations are expressed by simple formulas and therefore convenient to use in practical computations of the compensated effect of a change in the wage rate, see Corollaries 2 and 3 below.

Since the mid-1990s empirical labor supply analyses based on discrete choice models with random utility representations have become increasingly popular (Van Soest, 1995).[1] See Dagsvik et al. (2014) for a survey of discrete labor supply models. The random utility formulation asserts that the utility representation consists of a separable deterministic and a random component. The random component accounts for individual tastes that may vary across latent job attributes as well as across observationally identical individuals. The random component may also capture that tastes of a given individual are random to the individual him- or herself in the sense that they fluctuate in an unpredictable manner (to the individual) across identical "experimental" settings. This is due to the inability of the agent to assess a definite value of the alternatives once and for all. A major reason for the popularity of discrete labor supply models is that they are much more practical to apply than the conventional approach based on marginal calculus with a continuum of hours available. With the discrete choice approach, it is easy to deal with non-linear and non-convex economic budget constraints, and to apply rather general and flexible representations of preferences. Tax functions are typically piecewise linear (kinked) and in many countries the marginal tax rates are not uniformly increasing with taxable income and therefore imply non-convex budget constraints. For governments it is important to be able to assess the impact of changes in the whole tax system on labor supply, income distribution, and tax revenue. This can easily be done with discrete labor supply models without simplifying the tax system.

In the literature, basically two versions of discrete models of labor supply have been developed. The first version is based on a standard discrete choice approach (Van Soest, 1995) where labor supply is modeled similarly to the traditional textbook model. That is, the labor supply decision is based on maximization of utility with respect to consumption and hours of work, subject to a budget constraint. As in the textbook model agents are assumed to be free to choose any hours of work, subject to a finite set of feasible hours. As noted below, this feature is not in accordance with how labor markets are organized in most developed countries. The second version, which we refer to as the job choice model, was proposed by Dagsvik (1994).[2] In the job choice model, labor supply behavior is viewed as a job choice problem, where each job is characterized by job-specific hours of work, wage rate and latent non-pecuniary attributes. Furthermore, the set of feasible jobs is individual-specific and latent, and the set of feasible jobs with specific workloads is represented by a measure of job availability. The job choice model thus differs from the standard discrete labor supply model described above in that the individual's preferences depend explicitly on non-pecuniary job attributes, and in that the individual may be constrained in his or her choice of job and associated hours of work. This framework allows us to accommodate observed peaks in the hours of work distribution, which are interpreted as stemming from more jobs with full-time and part-time hours of work being available than jobs with other hours of work schedules. The reason for these humps at full-time and part-time hours is due to team work, government regulation of hours, and negotiations between trade unions and employers' organizations. It follows that the standard discrete labor supply model thus becomes a special case of the job choice model.

Instead of deriving analytic results for the Slutsky equation, one could alternatively conduct Monte Carlo (MC) simulations of marginal compensated effects.[3] However, it is important to derive analytic results for several reasons provided they are simple to use in computations, as is the case for the analytic results we have derived. First, analytic expressions reveal key qualitative features of the marginal compensated effects. For example, it turns out that in general the marginal compensated effect resulting from a wage increase differs from the marginal compensated effect resulting from a wage decrease. Second, analytic results are usually more precise than results based on MC simulations, because they are not plagued by simulation errors.

In Section 4 the empirical model for married women developed and estimated by Dagsvik and Strøm (2006) is applied to compute compensated wage elasticities at different levels of hypothetical wage rate, non-labor income, number of children in the household, and age of the woman. It turns out that with an increase in the wage rate the compensated and the uncompensated wage elasticities of mean hours of work are rather close, with the former being only slightly larger than the latter. Although theoretically the right and left compensated elasticities are different, as mentioned above, it turns out that the left and right compensated wage elasticities at the intensive margin are very close. However, at the extensive margin there are substantial differences between the right and left compensated wage elasticities.

The paper proceeds as follows. In Section 2 we present several versions of discrete labor supply models. In Section 3 we derive analytic formulas for marginal compensated wage effects and the corresponding Slutsky equation for discrete labor supply models. Section 4 contains counterfactual numerical-as well as empirical compensated and uncompensated labor supply wage elasticities based on the empirical model in Dagsvik and Strøm (2006).

[1] This type of labor supply model is now used by the Ministries of Finance in New Zealand and Norway, by the CPB (The Netherlands), the Melbourne Institute (Australia), the ZEW (Germany) and the IFS (UK), and by Joint Research Centre (JRC) of the European Commission for ex-ante analysis (i.e. before a reform is implemented).Some recent applications are found in Blundell and Shepard (2012), Peichel and Siegloch (2012), Bargain et al. (2014), Mastrogiacomo et al. (2017), and Coda Moscarolo et al. (2019).

[2] Empirical applications have been conducted by Dagsvik et al. (1988), Aaberge et al. (1995), Aaberge et al. (1999), Capeau et al. (2016), Dagsvik and Strøm (2006), Dagsvik et al. (2011), Dagsvik and Jia (2016), and Bezeredi et al. (2019). For a recent application of a related approach where there are restrictions on the hours available in the market, see Beffy et al. (2019).

[3] See Bargain et al. (2014), where uncompensated wage elasticities (only) in a van Soest-type model are computed numerically by simulations of 1 percent uniform increase in wage rates.

2. Versions of discrete labor supply models

2.1. The job choice model

In this section we provide a short review of the job choice model. As mentioned in the introduction, the job choice model developed by Dagsvik (1994) and further developed by Dagsvik and Strøm (2006) and Dagsvik and Jia (2016), allows the researcher to account for latent restrictions in the labor market. Such restrictions may explain why the distribution of hours of work typically shows peaks at full-time and part-time hours of work and that some workers face smaller sets (latent) of job opportunities than others. In this model the household derives utility from household consumption, leisure, and non-pecuniary latent job attributes.

Let $z = 1, 2, \ldots,$ be an indexation of the jobs and let $z = 0$ represent "not working". Let, respectively, w and y denote the agent's wage rate and non-labor income and $f(x, y)$ the function that transforms labor and non-labor income to income after tax, C. Thus, $C = f(hw, y)$. Let $U(C, h, z)$ denote the agent's utility function of disposable income, hours of work, and job type (job).

Assumption 1. The utility function $U(C, h, z)$ has the structure

$$U(C, h, z) = u(C, h) + \varepsilon(z)$$

where $u(\cdot)$ is a deterministic function that is strictly increasing in C and strictly decreasing in h, and $\{\varepsilon(z)\}$ are job-specific random taste shifters that are independent of $u(C, h)$ and i.i.d. across jobs and agents, with Gumbel c.d.f. $\exp(-e^{-x})$, for real x. The set of possible hours is finite, and the choice sets of available jobs are independent of the taste shifters.

The random taste shifters account for unobserved individual characteristics and unobserved job-specific attributes. The motivation for the Gumbel c.d.f. follows from Luce's choice axiom (Luce, 1959), which is a representation of probabilistic rationality (Luce, 1977).

One way of understanding labor market behavior in equilibrium is as a two-sided matching market where workers (firms) are looking for a suitable match with a firm (worker) (Dagsvik, 2000; Menzel, 2015; Dagsvik and Jia, 2018). Dagsvik and Jia (2018) have demonstrated that the equilibrium choice sets of job opportunities can be treated as if they were exogenous provided that the market is large.

Let D be the set of possible hours of work which we recall is assumed to be finite and let B(h) be the set of jobs with hours of work h that is available to the agent. The sets B(h), $h \in D$, are individual-specific and latent. Moreover, let θ be the number of jobs in B(h) and g(h) the proportion of jobs in B(h) with hours of work h. Thus, $\theta g(h)$ is the number of jobs (opportunity measure) with hours of work h in the latent set$B(h)$. The fractions $\{g(h)\}$ that represent restrictions on hours, may be interpreted as stemming from institutional regulations determined by the labor organizations, the firms or the authorities. Let $p(z)$ denote the probability that the agent shall choose job z and let $\varphi(h) = \varphi(h; w, y)$ denote the probability of supplying h hours given the wage rate and non-labor income (w, y). From Assumption 1 and well-known results in the theory of discrete choice, it follows that the probability that the agent shall choose job $z \in B(h)$ is given by

$$p(z) = P(U(f(hw, y), h, z) = \max_{x \in D} \max_{k \in B(x)} U(f(xw, y), x, k)) \tag{2.1}$$

$$= \frac{\exp(u(f(hw, y), h))}{\exp(u(f(0, y), 0)) + \sum_{x \in D \setminus \{0\}} \sum_{k \in B(h)} \exp(u(f(xw, y), x))}$$

$$= \frac{\exp(u(f(hw, y), h))}{\exp(u(f(0, y), 0)) + \sum_{x \in D \setminus \{0\}} \theta g(x) \exp(u(f(xw, y), x))}.$$

The last equality above follows from the fact that $u(f(xw, y), x)$ does not depend on k. Consequently, we obtain from (2.1) that the Marshallian probability $\varphi(h)$ that the agent shall choose to supply h hours of work is given by

$$\varphi(h) = \sum_{z \in B(h)} p(z) = \frac{\theta g(h) \exp(u(f(hw, y), h))}{\exp(u(f(0, y), 0)) + \sum_{x \in D \setminus \{0\}} \theta g(x) \exp(u(f(xw, y), x))}. \tag{2.2}$$

Thus, the utility maximization (with respect to hours of work) in the presence of these types of latent constraints can formally be viewed as an unconstrained maximization problem, namely the maximization of $V(h) \equiv v(h) + \eta_h$ with respect to h, where $\{\eta_h\}$ are independent and standard Gumbel distributed and where the structural part $v(h)$(say) is given by

$$v(h) = u(f(hw, y), h) + \log(\theta g(h)) \tag{2.3}$$

for$h > 0$, and $v(0) = u(f(0, y), 0)$ when h = 0. For h = 0, $\varphi(0)$ is obtained from (2.2) by replacing the numerator with exp(u(f(0, y),0)). Mathematically, then, the model given in (2.2) can be treated as if it were a standard discrete labor supply model with the function u replaced by the function v. The opportunity measure, $\theta g(h)$, is not directly observable in typical data sets. However, Dagsvik and Jia (2016) have demonstrated that u, θ, and g(h) can still be identified. They also show that the model is identified under specific assumptions, even in the case of unobserved heterogeneity of the wage equation.[4] Thus, by specifying a flexible functional form of the

[4] Remember that in empirical applications it is necessary to use a wage equation to predict wage rates for those who do not work and to take into account that the wage rate might be correlated with the preference for work.

systematic part u of the utility function and of the opportunity measure, up to a set of unknown parameters, this model can be readily estimated using maximum likelihood methods. The parameter θ may also capture unobserved fixed costs of working.

2.2. The standard discrete labor supply model

The standard discrete choice labor supply model was proposed by Van Soest (1995). This model was derived independently of the job choice model presented above, but it can also be viewed as a special case of the job choice model, obtained by assuming that there are no restrictions on choices apart from the budget constraint and the set of possible hours D. This means that the standard model follows by letting $\theta g(h) = 1$ so that the choice probability in (2.2) reduces to

$$\varphi(h) = \frac{\exp(u(f(hw,y),h))}{\sum_{x\in D}\exp(u(f(xw,y),x))} \tag{2.4}$$

Recall that the hypothesis underlying (2.4) is that the agents are free to choose any of the hours in the choice set D.

2.3. The multi-sectoral job choice model

Dagsvik and Strøm (2006) extended the job choice model to a two-sectoral version. The two-sector model, with private and public sectors, is of specific interest because trade unions are more influential in the public sector than in the private one, and therefore most of the regulations occur in the public sector and consequently hours are more restricted in the public than in the private sector. The model structure in the case with more than two sectors is analogous. In the multi-sectoral job choice model, preferences are represented by a utility function $U(C,h,j,z)$ where j indexes sector.

Assumption 2. The utility function $U(C,h,j,z)$ has the structure

$$U(C,h,j,z) = u(C,h) + \varepsilon_j(z)$$

where $\{\varepsilon_j(z)\}$ are job-specific random taste shifters that are independent of $u(C,h)$ and are i.i.d. across job sectors and agents, with Gumbel c.d.f. $\exp(-e^{-x})$, for real x. The set of possible hours is finite, and the choice sets of available jobs are independent of the taste shifters.

In the multi-sectoral case, as in the one-sector case, Dagsvik and Strøm (2006) show that Assumption 2 implies that the probability of working h hours in sector j becomes

$$\varphi_j\left(h\right) = \frac{\theta_j g_j(h)\exp\left(u\left(f\left(hw_j,y\right),h\right)\right)}{\exp\left(v\left(0,y\right)\right) + \sum_r\sum_{x\in D\setminus\{0\}}\theta_r g_r(x)\exp(u(f(xw_r,y),x)} \tag{2.5}$$

where w_j is the wage rate in sector j and $\theta_j g_j(h)$ is the sector j-specific opportunity measure, which has a similar interpretation as in the one-sector case. The probability of not working follows from (2.5) by replacing the numerator in (2.5) by $\exp(u(f(0,y),0))$. Just as in the one-sector case, the multi-sectoral job choice model can formally be viewed as a standard discrete labor supply model by defining the utility $V_j(h) \equiv v_j(h) + \eta_{jh}$ of working h hours in sector j where $\{\eta_{jk}\}$ are independent and Gumbel distributed, $v_j(h)$ is

$$v_j(h) = u\left(f\left(hw_j,y\right),h\right) + \log\left(\theta_j g_j(h)\right)$$

for positive h and $v(0) = u(f(0,y),0)$. We realize that the standard discrete labor supply model and the one-sectoral job choice become special cases of the multi-sectoral job choice model (2.5). This is convenient for our subsequent analysis because it allows a unified treatment.

The two-sectoral job choice model implies two extensive margins: to work or not work, and to work in the public or the private sector. It has been applied to conduct welfare analysis by Dagsvik et al. (2009).

3. Compensated effects and slutsky equations

3.1. The classical slutsky equation

In the traditional textbook case, where the set of available hours of work D is a continuum, the substitution effect can be visualized as a move along an indifference curve. To calculate the marginal compensated effects from a change in the wage rate in the textbook case, one can apply the Slutsky equation. To review the classical Slutsky equation, let$h(w,y)$denote the Marshallian labor supply of hours of work, as a function of the wage rate and non-labor income (w, y), and let $h^c(w,u)$ denote the corresponding compensated (Hicksian) labor supply function, where u is the utility level. At optimum, $h^c(w,u) = h(w,e(w,u))$, where $e(w,u)$ is the expenditure function needed to keep utility at the level u. Using that at optimum non-wage income y is equal to $e(w,u)$, and applying Shephard's lemma, the Slutsky equation follows by differentiating through the expression above with respect to w, which yields

$$\frac{\partial h(w,y)}{\partial w} = \frac{\partial h^c(w,u)}{\partial w} + h(w,y)\frac{\partial h(w,y)}{\partial y}.$$

This equation allows one to compute the marginal compensated labor supply wage effects (which are unobservable) from the corresponding marginal Marshallian labor supply wage effects. In traditional labor supply models, where hours of work are often determined by a closed-form expression, it is straightforward to calculate the compensated marginal effects. As noted above, in discrete labor supply models the labor supply function is stochastic and cannot be expressed on closed form. Therefore, another approach is called for which we shall discuss below.

3.2. Marginal compensated wage effects in discrete labor supply models and the slutsky equation

We shall now consider marginal compensated wage effects in discrete labor supply models. Our approach, which is based on Dagsvik and Karlström (2005), yields analytic results for the marginal compensated labor supply wage effects and the Slutsky equation for such models. To the best of our knowledge, Slutsky equations for discrete labor supply models have not been obtained previously.

Consider a setting where the wage rate changes from the initial ex-ante value w to the ex-post value $\widetilde{w}$ (say). Let $Q^c(x,h) = Q^c(x, h; w, y, \widetilde{w})$ be the joint compensated probability of choosing x hours of work ex-ante and h hours of work ex-post, where y is the ex-ante non-labor income (exogenous), given that the ex-ante and ex-post utility levels are equal. Let $P^c(h; w, \widetilde{w}, y)$ be the compensated probability of choosing h hours of work ex-post, given that the indirect utility is kept fixed and equal to the ex-ante level. It follows that

$$P^c\left(h; w, \widetilde{w}, y\right) = \sum_{x \in D} Q^c\left(x, h\right) \tag{3.1}$$

We wish to compute the compensated marginal effect of the choice probability of working h hours. To this end it turns out that one needs to introduce left and right derivatives of the compensated probability of working h hours, defined by

$$\frac{\partial^+ \varphi^c(h)}{\partial w} = \lim_{\widetilde{w}\downarrow w} \frac{P^c(h; w, \widetilde{w}, y) - \varphi(h; w, y)}{\widetilde{w} - w} \tag{3.2}$$

and

$$\frac{\partial^- \varphi^c(h)}{\partial w} = \lim_{\widetilde{w}\uparrow w} \frac{P^c(h; w, \widetilde{w}, y) - \varphi(h; w, y)}{\widetilde{w} - w}. \tag{3.3}$$

In (3.2) $\widetilde{w} - w > 0$, and it approaches zero from above, whereas in (3.3) $\widetilde{w} - w < 0$, and it approaches zero from below. The formula in (3.2) is the right marginal compensated effect of the probability of working h hours with respect to the wage rate. This formula is relevant for computing the compensated marginal effect of an *increase* in the wage rate. The formula in (3.3) is the corresponding left marginal compensated effect, which is relevant for computing the marginal compensated effect of a *decrease* in the wage rate. Remember that the derivative $\partial\varphi^c(h)/\partial w$ exists only if the left and right derivatives are equal. Surprisingly, this derivative does not always exist. Let Z be a real number and define Z_+ by $Z_+ = \max(Z, 0)$. Furthermore, let

$$\gamma(h) = \frac{\partial v(h)/\partial w}{\partial v(h)/\partial y} = \frac{f_1'(hw,y)h}{f_2'(hw,y)} \tag{3.4}$$

where we recall that $f(hw, y)$ is the function that transforms labor and non-labor income to income after tax.

In what follows we shall use the term discrete labor supply model, which includes the job choice model as well as the special case of the standard discrete labor supply model. We get the following result.

Theorem 1. *Under Assumption 1 the compensated right and left wage elasticities of the p.d.f. of hours of work in the discrete labor supply model are given by*

$$\frac{\partial^+ \log \varphi^c(h)}{\partial \log w} = w\frac{\partial v(h)}{\partial y}\sum_{x\in D}\varphi(x)(\gamma(h)-\gamma(x))_+ - w\sum_{x\in D}\frac{\partial v(x)}{\partial y}\varphi(x)(\gamma(x)-\gamma(h))_+$$

and

$$\frac{\partial^- \log \varphi^c(h)}{\partial \log w} = w\sum_{x\in D}\frac{\partial v(x)}{\partial y}\varphi(x)(\gamma(h)-\gamma(x))_+ - w\frac{\partial v(h)}{\partial y}\sum_{x\in D}\varphi(x)(\gamma(x)-\gamma(h))_+$$

for $h \in D$ where $v(h)$ and $\gamma(h)$ are, respectively, given in (2.3) and (3.1). Theorem 1 is a special case of Theorem 2 (Appendix A) with Proof given in Appendix B. In contrast to the traditional textbook case, the formulas given in Theorem 1 express marginal aggregate compensated effects. The corresponding marginal uncompensated (Marshallian) wage elasticity is given by

$$\frac{\partial \log \varphi(h)}{\partial \log w} = w\left(\frac{\partial v(h)}{\partial w} - \sum_{x\in D}\frac{\partial v(x)}{\partial w}\varphi\left(x\right)\right) = w\left(\frac{\partial v(h)}{\partial y}\gamma\left(h\right) - \sum_{x\in D}\frac{\partial v(x)}{\partial y}\gamma\left(x\right)\varphi\left(x\right)\right) \tag{3.5}$$

which is straightforward to verify.

Let $\tilde{h}$ denote realized hours of work, that is, $\tilde{h}$ is a draw from the p.d.f. $\varphi(h)$. From Theorem 1 and (3.5) the next result follows.

Corollary 1. *Under Assumption 1 the compensated right and left, and uncompensated wage elasticities of the p.d.f. of hours of work in the discrete labor supply model can be expressed as*

$$\frac{\partial^+ \log \varphi^c(h)}{\partial \log w} = w\frac{\partial v(h)}{\partial y}E\left(\gamma\left(h\right) - \gamma\left(\tilde{h}\right)\right)_+ - wE\left(\frac{\partial v\left(\tilde{h}\right)}{\partial y}\left(\gamma\left(\tilde{h}\right) - \gamma\left(h\right)\right)_+\right)$$

$$\frac{\partial^- \log \varphi^c(h)}{\partial \log w} = wE\left(\frac{\partial v\left(\tilde{h}\right)}{\partial y}\left(\gamma\left(h\right) - \gamma\left(\tilde{h}\right)\right)_+\right) - w\frac{\partial v(h)}{\partial y}E\left(\gamma\left(\tilde{h}\right) - \gamma\left(h\right)\right)_+$$

and

$$\frac{\partial \log \varphi(h)}{\partial \log w} = w\frac{\partial v(h)}{\partial y}\gamma\left(h\right) - wE\left(\frac{\partial v\left(\tilde{h}\right)}{\partial y}\gamma\left(\tilde{h}\right)\right).$$

The formulas in Corollary 1 show that with a simulation routine that generates hours of work $\tilde{h}$ from $\varphi(h)$ then it becomes very easy to compute the compensated and uncompensated wage elasticities by replacing the expected values by the corresponding simulated averages.

Corollary 2 below follows readily from (3.5) and Theorem 1.

Corollary 2. (*Slutsky equation for the p. d. f. of hours of work*).

Under Assumption 1 *the right Slutsky equation for the labor supply elasticities of the* choice probabilities of hours with respect to changes in the wage rate is given by

$$\frac{\partial \log \varphi(h)}{\partial \log w} = \frac{\partial^+ \log \varphi^c(h)}{\partial \log w} - w\sum_{x\in D}\left(\frac{\partial v(x)}{\partial y} - \frac{\partial v(h)}{\partial y}\right)\varphi\left(x\right)\min(\gamma(h), \gamma(x)) \quad \left(i\right)$$

and the left Slutsky equation is given by

$$\frac{\partial \log \varphi(h)}{\partial \log w} = \frac{\partial^- \log \varphi^c(h)}{\partial \log w} - w\sum_{x\in D}\left(\frac{\partial v(x)}{\partial y} - \frac{\partial v(h)}{\partial y}\right)\varphi\left(x\right)\max(\gamma(h), \gamma(x)). \quad \left(ii\right)$$

The relations given in Theorem 1 and Corollary 2 show that the right marginal compensated wage effect differs from the left marginal compensated wage effect. From the equation for the right marginal effect in (*i*) it follows when $h > 0$ that the income effect is given by

$$-w\sum_{x\in D}\left(\frac{\partial v(h)}{\partial y} - \frac{\partial v(x)}{\partial y}\right)\varphi\left(x\right)\min(\gamma(h), \gamma(x)),$$

which can be positive or negative, and even equal to zero. From Corollary 2 it follows that

$$\frac{\partial \log \varphi(0)}{\partial \log w} = \frac{\partial^+ \log \varphi^c(0)}{\partial \log w} \text{ and } \frac{\partial \log \varphi(0)}{\partial \log w} = \frac{\partial^- \log \varphi^c(0)}{\partial \log w} - w\sum_{x\in D}\left(\frac{\partial v(x)}{\partial y} - \frac{\partial v(0)}{\partial y}\right)\gamma\left(x\right)\varphi\left(x\right)$$

We note that at the extensive margin there is no income effect in the expression for the right marginal compensated effect. This is in accordance with the classical textbook case. In contrast, there is an income effect in the expression for the left marginal compensated effect. The reason for this is that, with a wage decrease, wage income is lost in the transition from working to not working and this loss therefore must be compensated.

From Corollary 2 it also follows readily that

$$\frac{\partial^- \log \varphi^c(h)}{\partial \log w} - \frac{\partial^+ \log \varphi^c(h)}{\partial \log w} = w\sum_{x\in D}\left|\gamma\left(h\right) - \gamma\left(x\right)\right|\left(\frac{\partial v(h)}{\partial y} - \frac{\partial v(x)}{\partial y}\right)\varphi\left(x\right) \tag{3.6}$$

The expression in (3.6) shows that the sign of the difference between the right and left marginal compensating effects may be positive or negative, as well as zero.

To gain some understanding of why the right and left marginal compensated effects are different we shall discuss the binary version of the standard discrete labor supply model. with the options “not working” (state 0) versus “working full time” (state 1), and where the wage changes from w to $\tilde{w}$. The agent has preferences over consumption and hours of work (C, h) with utility function $U(C, h) =$

$U(C, h; \varepsilon_h)$ that depends on a random taste shifter ε_h. We assume that $U(C, h; \varepsilon_h)$ is strictly increasing in C with probability 1. Let $h = 1$ represent full time workload. For simplicity we only consider the case without taxes since the argument in the general case is similar. Thus, the budget constraint is $C = w + y$ where w is the wage and y is nonlabor income. Let $\widetilde{w}$ be the ex post wage, Q^c_{jk} denote the probability being in state j ex ante and in state k ex post, $j, k = 0, 1$, and Y_{jk} the corresponding (ex post) expenditure.

Suppose first that $\widetilde{w} > w$. Consider transitions from state 0 to state 1. In this case there are income effects, with Y determined by $U(y, 0) = U(\widetilde{w} + Y_{01}, 1)$. However, it turns out that the income effects vanish when aggregating. To realize this, note that in this case

$$Q^c_{01} = P(U(w + y, 1) < U(y, 0) = U(\widetilde{w} + Y_{01}, 1) > U(Y_{01}, 0))$$

implying that $Y_{01} < y$, since $U(y, 0) > U(Y_{01}, 0)$ with probability 1. Thus, Q^c_{01} reduces to

$$Q^c_{01} = P(U(w + y, 1) < U(y, 0) = U(\widetilde{w} + Y_{01}, 1)) = P(U(w + y, 1) < U(y, 0) < U(\widetilde{w} + y, 1))$$

$$= P(U(y, 0) < U(\widetilde{w} + y, 1)) - P(U(y, 0) < U(w + y, 1))$$

which we note are the difference between the ex post and ex ante probability of working.

Furthermore,

$$Q^c_{11} = P(U(y, 0) < U(w + y, 1) = U(\widetilde{w} + Y_{11}, 1) > U(Y_{11}, 0))$$

implying that $Y_{11} = w - \widetilde{w} + y < y$. Since $U(w - \widetilde{w} + y, 0) < U(y, 0)$ with probability 1, Q^c_{11} reduces to

$$Q^c_{11} = P(U(y, 0) < U(w + y, 1)).$$

Hence, the compensated probability of choosing state 1 ex post becomes

$$P^c(1; w, \widetilde{w}, y) = Q^c_{01} + Q^c_{11} = P(U(y, 0) < U(\widetilde{w} + y, 1)) \tag{3.7}$$

which equals the uncompensated probability of choosing state 1 ex post.

Consider next the case where $\widetilde{w} < w$. In this case

$$Q^c_{01} = P(U(w + y, 1) < U(y, 0) = U(\widetilde{w} + Y_{01}, 1) > U(Y_{01}, 0)) = 0$$

because it is not possible that both $U(y, 0) > U(Y_{01}, 0)$and $U(w + y, 1) < U(\widetilde{w} + Y_{01}, 1)$. Furthermore, $Y_{11} = w - \widetilde{w} + y$ so that

$$Q^c_{11} = P(U(y, 0) < U(w + y, 1) = U(\widetilde{w} + Y_{11}, 1) > U(Y_{11}, 0))$$

$$= P(U(w + y, 1) > U(w - \widetilde{w} + y, 0)).$$

The last result is intuitive because when the worker does not change state she or he needs to be compensated after a wage decrease. Hence

$$P^c(1; w, \widetilde{w}, y) = Q^c_{01} + Q^c_{11} = Q^c_{11} = P(U(w + y, 1) > U(w - \widetilde{w} + y, 0)). \tag{3.8}$$

We note that there is a fundamental difference between (3.7) and (3.8). In (3.7) the utility of working depends on consumption $\widetilde{w}+ y$ and the utility of not working depends on consumption y. In contrast, in (3.8) the utility of working depends on consumption $w+ y$ whereas the utility of not working depends on consumption $w - \widetilde{w} + y$. Accordingly, the left marginal compensated effect differs from the right marginal compensated effect.

Further intuition might be obtained by considering the special case where

$$U(C, 1) = v_1(C) + \varepsilon_1 \text{ and } U(C, 0) = v_0(C) + \varepsilon_0$$

and where F is the c. d. f. of $\varepsilon_0 - \varepsilon_h$. Then if $\widetilde{w} > w$ we get from (3.7) that

$$P^c(1; w, \widetilde{w}, y) = F(v_1(\widetilde{w}, y) - v_0(y)) \tag{3.9}$$

and if $\widetilde{w} < w$ we get from (3.8) that

$$P^c(1; w, \widetilde{w}, y) = F(v_1(w, y) - v_0(y_h)). \tag{3.10}$$

From (3.9) we get that the corresponding right marginal compensated wage effect is

$$\frac{\partial^+ \varphi^c(h)}{\partial w} = F'(v_1(w, y) - v_0(y)) \frac{\partial v_1(w, y)}{\partial w}$$

whereas from (3.10) we get that the left marginal compensated wage effect is

$$\frac{\partial^- \varphi^c(h)}{\partial w} = -F'(v_1(w, y) - v_0(y)) \frac{\partial v_0(y)}{\partial y} \cdot \frac{\partial y_h}{\partial w}$$

$$= F'(v_1(w,y) - v_0(0;y))\frac{\partial v_0(y)}{\partial y}\cdot\frac{\partial v_1(w,y)}{\partial w}\Big/\frac{\partial v_1(w,y)}{\partial y}.$$

It follows from the expressions above that

$$\frac{\partial^+\varphi^c(1)/\partial w}{\partial^-\varphi^c(1)/\partial w} = \frac{\partial v_1(w,y)/\partial y}{\partial v_0(y)/\partial y}$$

which in general differs from 1.

From Corollary 2 the Slutsky equations for the mean hours of work follow readily and are given in the next Corollary.

Corollary 3. (*Slutsky equations for the mean hours of work*)

Let $\widetilde{h}$ and $\widetilde{h}'$ be independent realizations of hours of work generated by $\varphi(h)$. Then, under Assumption 1, *the right and left Slutsky equations for the mean hours of work are given by*

$$\frac{\partial E\widetilde{h}}{\partial w} = \frac{\partial^+ E\widetilde{h}^c}{\partial w} - Cov\left(\widetilde{h}, -\frac{\partial v\left(\widetilde{h}\right)}{\partial y}\min\left(\gamma\left(\widetilde{h}\right),\gamma\left(\widetilde{h}'\right)\right)\right) + Cov\left(\widetilde{h}', -\frac{\partial v\left(\widetilde{h}\right)}{\partial y}\min\left(\gamma\left(\widetilde{h}\right),\gamma\left(\widetilde{h}'\right)\right)\right)$$

and

$$\frac{\partial E\widetilde{h}}{\partial w} = \frac{\partial^- E\widetilde{h}^c}{\partial w} - Cov\left(\widetilde{h}, -\frac{\partial v\left(\widetilde{h}\right)}{\partial y}\max\left(\gamma\left(\widetilde{h}\right),\gamma\left(\widetilde{h}'\right)\right)\right) + Cov\left(\widetilde{h}', -\frac{\partial v\left(\widetilde{h}\right)}{\partial y}\max\left(\gamma\left(\widetilde{h}\right),\gamma\left(\widetilde{h}'\right)\right)\right).$$

From Corollary 3 the next result follows.

Corollary 4. *Under* Assumption 1 *we have that*

$$\frac{\partial E\widetilde{h}}{\partial w} \le \frac{\partial^+ E\widetilde{h}^c}{\partial w} \text{ and } \frac{\partial E\widetilde{h}}{\partial w} \le \frac{\partial^- E\widetilde{h}^c}{\partial w}.$$

The Proof of the inequalities in Corollary 4 runs as follows. Note first that $-\partial v(h)/\partial y$ and $\gamma(h)$ are increasing functions of h and second that the correlation between $\widetilde{h}$ and $-\min(\gamma(\widetilde{h}),\gamma(\widetilde{h}'))\partial v(\widetilde{h})/\partial y$ is higher than the correlation between $\widetilde{h}'$ and $-\partial v(\widetilde{h})/\partial y\min(\gamma(\widetilde{h}), \gamma(\widetilde{h}'))$ because $\widetilde{h}$ and $\widetilde{h}'$ are independent. Hence, the income effect

$$-Cov\left(\widetilde{h}, -\frac{\partial v\left(\widetilde{h}\right)}{\partial y}\min\left(\gamma\left(\widetilde{h}\right),\gamma\left(\widetilde{h}'\right)\right)\right) + Cov\left(\widetilde{h}', -\frac{\partial v\left(\widetilde{h}\right)}{\partial y}\min\left(\gamma\left(\widetilde{h}\right),\gamma\left(\widetilde{h}'\right)\right)\right) < 0$$

from which the inequalities in Corollary 4 follow. This property is analogous to the standard textbook case given that leisure is a normal good. We would expect the difference between the two covariances in most cases to be small and so we would expect the income effect above to be small. The implication is that we would expect the uncompensated and the compensated mean hours wage elasticities to be rather similar.

Because of non-linearity and unobserved heterogeneity at the micro level, the compensated and uncompensated labor supply elasticities cannot be expressed in a simple way by structural parameters. Also, it can be shown that even when the model is a continuous multinomial logit model, as in Dagsvik (1994), the difference between the right and left marginal wage effects does not disappear.

Remember that although the formulas above are based on a multinomial logit formulation they can easily be extended to the mixed logit case by allowing some parameters to be random.

4. Compensated and uncompensated labor supply wage elasticities: counterfactuals and empirical results

To illustrate how the compensated and uncompensated labor supply wage elasticities differ across individual characteristics, wage, and non-labor income levels, we have calculated labor supply wage and income elasticities at different hypothetical levels of wage rates and non-labor incomes, given the estimates of the empirical two-sector supply model for married women (Dagsvik and Strøm, 2006). We also report the mean values of compensated and uncompensated elasticities for a representative sample used in the estimation of the model. Details about data, tax functions, and estimates of the model are given in Appendix C in the online supplementary section of this paper. All values below refer to 1994–1995. Since then the nominal wage level has increased substantially. Wages in the

Table 1
Right compensated and uncompensated mean wage elasticities with respect to an overall wage increase for the sample used in estimating the model. Married women, Statistics Norway, 1994.

Elasticities	Probability of working	Conditional hours, given working	Unconditional hours
Uncompensated	0.315	0.372	0.697
Compensated	0.315	0.401	0.716

Table 2
Right compensated and uncompensated wage elasticities with respect to an overall wage increase. Married woman aged 30, no children, wage rate NOK 70.

Non-labor income	Direct elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Pub.	Priv.	All	Pub.	Priv.	All	Pub.	Priv.
70,000	Uncompensated	0.010	0.047	−0.140	0.255	0.236	0.330	0.265	0.283	0.190
	Compensated	0.010	0.054	−0.170	0.307	0.284	0.397	0.317	0.339	0.226
100,000	Uncompensated	0.044	0.090	−0.139	0.325	0.303	0.407	0.368	0.393	0.269
	Compensated	0.044	0.096	−0.162	0.362	0.337	0.454	0.406	0.433	0.292
200,000	Uncompensated	0.164	0.219	−0.042	0.419	0.396	0.498	0.583	0.615	0.456
	Compensated	0.164	0.221	−0.050	0.431	0.408	0.512	0.595	0.629	0.462

private sector were the highest ones.[5]

The deterministic part of the utility function used in the estimation is given by

$$u\left(f\left(hw_j, y\right)\right), h, X\right) = \frac{\left(10^{-4}\left(f\left(hw_j, y\right) - C_0\right)\right)^{\alpha_1} - 1}{\alpha_1}\left[\alpha_2 + \alpha_9 \frac{(1 - h/3640)^{\alpha_3} - 1}{\alpha_3}\right] + Xb\frac{(1 - h/3640)^{\alpha_3} - 1}{\alpha_3} \tag{4.1}$$

where C_0 is the subsistence consumption level and X is a vector of observed characteristics, consisting of the number of children 0–6, 7–17, log age, and log age squared. The term $1 - h/3460$ represents normalized leisure after deducting time for rest and sleep. Non-labor income, y, is the sum of capital income after tax, child allowances, and the after-tax income of the husband. The parameter estimates (see Dagsvik and Strøm, 2006) imply that the utility function is quasi-concave, with a significant negative interaction term for leisure and consumption.

4.1. Overall wage change and aggregate elasticities using sample data

Here we report mean wage elasticities based on the same sample that was used to estimate the model. In Table 1 we report the right compensated and uncompensated mean elasticities of the mean hours of work and the probability of working with respect to an *overall increase* in the wage rates, as well as the corresponding mean conditional elasticities, given participation. Recall that individuals can choose to work in either the public or the private sector. The sector-specific changes are, however, not displayed here.

According to Theorem 2 (Appendix A), the uncompensated and the right compensated elasticities of working are equal. From Table 1 we note that the right compensated elasticity of mean total hours of an overall wage change, conditional on working (and hence also in the unconditional case), exceeds the corresponding uncompensated elasticity. However, the differences between the marginal compensated and uncompensated wage elasticities of the mean hours of work are not large. This means that the income effect is relatively small.

Attanasio et al. (2018) have calculated compensated labor supply wage elasticities based on US micro data (1980–2012). Although their model (a standard textbook labor supply model), sample, and aggregation procedure are different from ours, they obtain that the median in the distribution of the compensated wage elasticity, conditional on working, is equal to 0.54. In our case the aggregate compensated elasticity of mean hours of work, conditional on working, is equal to 0.40. However, the corresponding unconditional elasticity, accounting for the three extensive margins (working at all, working in the public sector, and working in the private sector) is 0.71. Bargain et al. (2014), who estimate a standard discrete labor supply model based on data from around year 2000 for 17 European countries, report only uncompensated labor supply own wage elasticities, which are not very different from the corresponding aggregate uncompensated elasticities we have obtained. For a large group of countries their estimates of the mean uncompensated own wage elasticities of hours of work are in the range of 0.2–0.4. They also report extremely small income elasticities, which is also what we get.

[5] The average nominal wage per hour for married women in 1994/1995 was around NOK 90–100, while the nominal average in December 2019 was around NOK 290.

Table 3
The selected household types.

Type no.	Age	Wage rate	No children	Non-labor income		
1–3	30	70	0	70,000	100,000	200,000
4–6	30	200	0	70,000	100,000	200,000
7–9	30	300	0	70,000	100,000	200,000
10–12	30	70	2	70,000	100,000	200,000
13–15	30	200	2	70,000	100,000	200,000
16–18	30	300	2	70,000	100,000	200,000
19–21	40	70	0	70,000	100,000	200,000
22–24	40	200	0	70,000	100,000	200,000
25–27	40	300	0	70,000	100,000	200,000
28–30	40	70	2	70,000	100,000	200,000
31–33	40	200	2	70,000	100,000	200,000
34–36	40	300	2	70,000	100,000	200,000

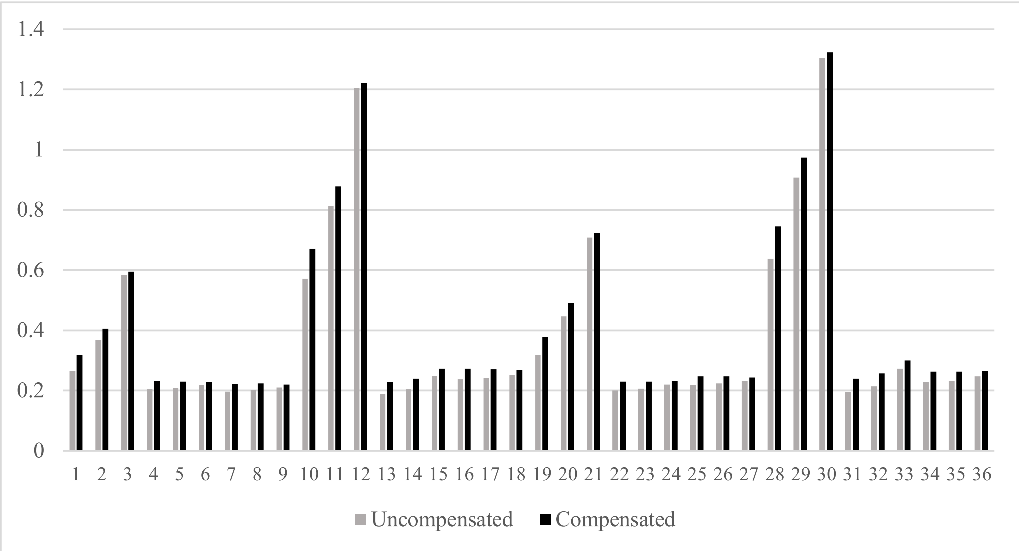

Fig. 1. Right compensated and uncompensated wage elasticities of total unconditional expected hours for an overall wage increase.

4.2. *An overall wage increase: labor supply elasticities for a specific household*

For simplicity, we consider the case where the wage rates for each sector are equal for all women. We report the marginal uncompensated and compensated wage elasticities of working at all, working in the public and the private sectors, conditional mean hours given participation, conditional mean hours given that the individual works in the respective sectors, and the respective unconditional mean hours (Corollaries 5 and 7 in Appendix A). With an overall wage change there are income effects related to the choice of working in the respective sectors, in contrast to the case with sector-specific wage changes (reported below).

In Table 2 we report elasticities for a married woman with wage rate NOK 70 and with three levels of non-labor income. This wage rate is around 20–30 percent below the average wage rate in 1994/1995.

All the compensated elasticities of mean hours of work conditional on working, as well as the mean unconditional hours, exceed the corresponding uncompensated ones, implying non-zero income effects. But these income effects are small, and they decline with the level of non-labor income. Moreover, the higher the non-labor income is, the lower is the probability of participation and the higher the wage elasticities. See Appendix E in the online supplementary section for more results.

4.3. *Selected household types*

We have computed wage and income elasticities for selected groups of married women who differ with respect to levels of wage rates (low; medium; and high), non-labor income (low, high, and high), number of children (none or two), and age (30 and 40). In total we have 36 different cases, displayed in Table 3 below. Of special interest are the groups with low wage rates, namely groups 1–3 (age 30, no children), 10–12 (age 30, two children 0–6), 19–21 (age 40, no children), and 28–30 (age 40, two children 7–17). The three cases in each of these groups have an increasing level of non-labor income. The probabilities of participation at all and participation in the two sectors, and the mean hours of work for these 36 cases are given in Appendix D in the online supplementary section. Numerical

Table 4
Right compensated and uncompensated wage elasticities. Married woman aged 30, no children, hourly wage NOK 70. Increase in the wage rate in the public sector.

Income		Probability of working			Conditional hours			Unconditional hours		
		Total	Public (direct)	Private (cross)	Total	Public (direct)	Private (cross)	Total	Public (direct)	Private (cross)
70,000	Uncompensated	0.008	1.442	−5.753	0.221	0.236	0.000	0.229	1.679	−5.753
	Compensated	0.008	1.442	−5.753	0.252	0.275	0.000	0.260	1.717	−5.753
100,000	Uncompensated	0.035	1.341	−5.094	0.281	0.303	0.000	0.316	1.643	−5.094
	Compensated	0.035	1.341	−5.094	0.303	0.330	0.000	0.338	1.671	−5.094
200,000	Uncompensated	0.131	1.184	−3.839	0.359	0.396	0.000	0.490	1.580	−3.839
	Compensated	0.131	1.184	−3.839	0.367	0.405	0.000	0.498	1.589	−3.839

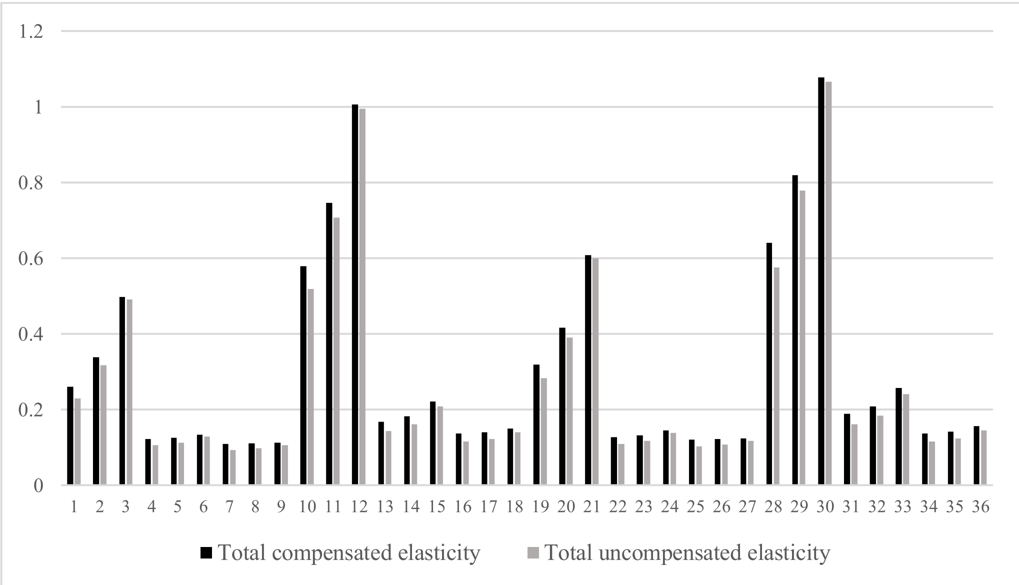

Fig. 2. Total unconditional expected hours elasticities related to a wage increase in the public sector.

values for all elasticities and for all cases shown below are given in Appendices E–J in the online supplementary section.

4.4. *Wage elasticities of mean hours of work within each household group with respect to an overall wage change*

In Fig. 1 we report the elasticities of mean hours (unconditional) of work for each of the 36 different household types. These wage elasticities are relatively small, with 12 exceptions. These 12 groups have low wage rates. Among them the wage elasticities are higher if they have children and/or if the non-labor income is high. These results clearly demonstrate that heterogeneity matters.

Of special interest here is that the compensated wage elasticities of mean hours exceed the corresponding uncompensated elasticities. Thus, there are income effects, but, as seen from Fig. 1, they are quite small in all cases.

4.5. *Sector-specific wage increases and elasticities for the 36 selected households*

In Table 4 we report uncompensated and right marginal compensated wage elasticities, due to an increase in the wage rate of the public sector only. In accordance with the theoretical results in Section 3, the compensated and uncompensated elasticities related to working at all and working in a specific sector are equal. This is true for the direct as well as the cross-elasticities. The compensated direct elasticities of mean hours are always higher than the corresponding uncompensated elasticities. Again, the income effects are small.

The uncompensated cross-wage elasticity of mean conditional hours of work, conditional on working in a specific sector, is equal to zero. This follows from the fact that when $j \neq k$ we have

$$E\left(\tilde{h}_k \middle| \tilde{h}_k > 0\right) = \frac{\sum_x x\varphi_k(x)}{\sum_x \varphi_k(x)} = \frac{\sum_x x \exp(v_k(x))}{\sum_x \exp(v_k(x))}$$

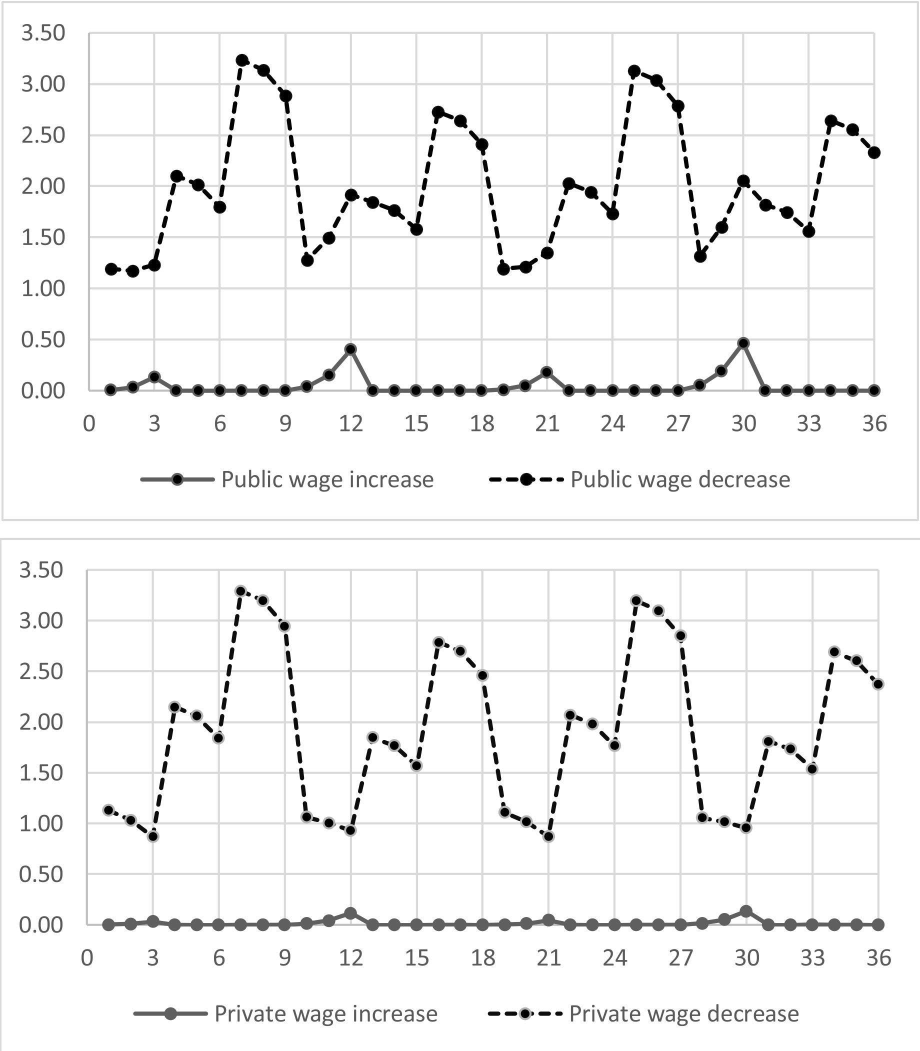

Fig. 3. Compensated elasticities of the probability of working.

which does not depend on w_j.

In Fig. 2 we report the wage elasticities of the unconditional mean of total hours with respect to an increase in the public wage rate for the selected 36 household types. "Total" means that the cross-effects are accounted for. The compensated elasticities exceed the uncompensated elasticities, but the difference is almost negligible. The elasticities are small, around 0.2, except for the women with low wages.

4.6. *Sector-specific wage increases versus wage decreases and elasticities*

In Appendix H (wage decrease in the public sector) and Appendix I in the online supplementary section (wage decrease in the private sector) we report left wage elasticities for all 36 household types. Fig. 3 yields the total wage elasticities of the probability of working (extensive margin). "Total" means that cross-effects are accounted for. Fig. 4 yields the elasticities of the conditional mean hours (intensive margin) and, finally, Fig. 5 yields the elasticities of the unconditional mean hours of work.

Note that at the extensive margin the left total wage elasticities are much higher than the right total wage elasticities. At the intensive margin the differences between left and right total wage elasticities are almost negligible for wage changes in the public sector and somewhat larger, but not much, for wage changes in the private sector.

5. Conclusions

By using the results of Dagsvik and Karlström (2005) we have derived analytic formulas and Slutsky equations for the compensated

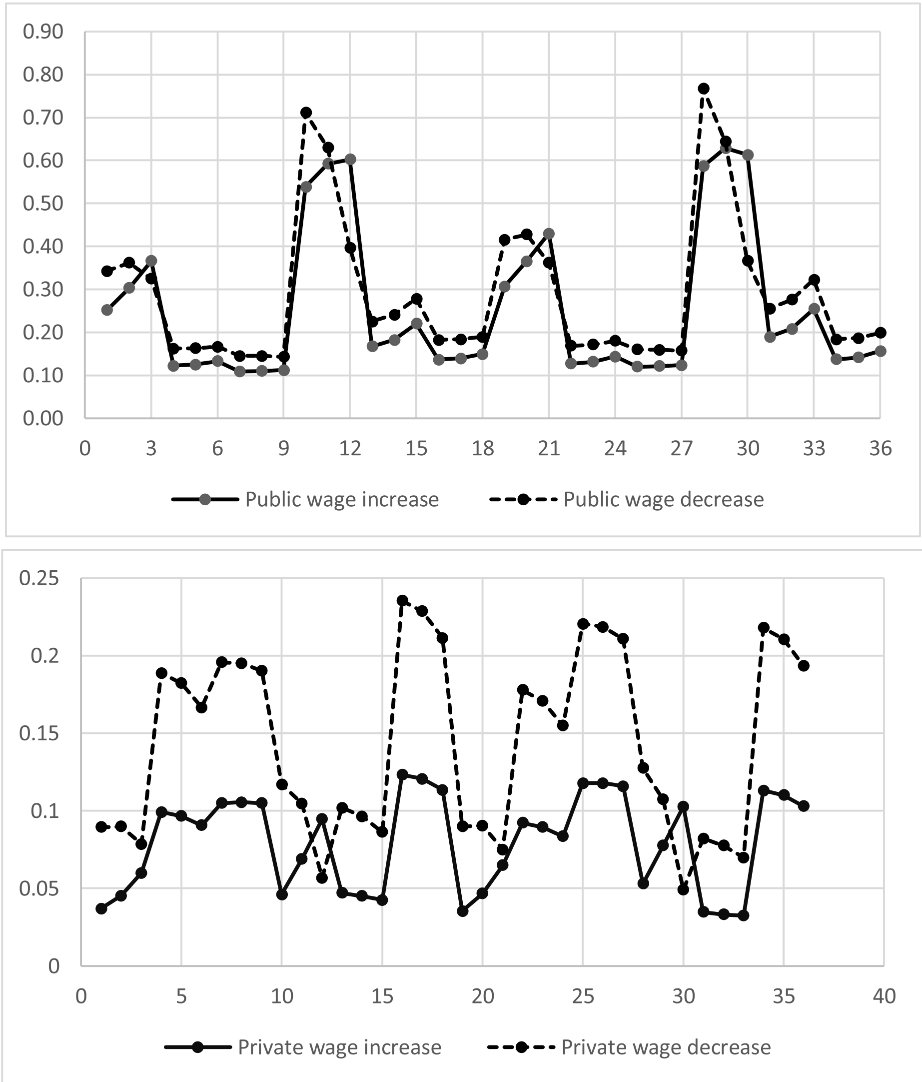

Fig. 4. Total compensated wage elasticities of conditional expected hours.

labor supply wage elasticities in discrete labor supply models. The Slutsky equation enables researchers to calculate compensated wage elasticities from the corresponding uncompensated wage elasticities. The Slutsky equation we have obtained differs in important ways from the traditional one in that the compensated labor supply elasticities with respect to a wage increase differs from the compensated wage elasticities with respect to a wage decrease.

In the selected numerical counterfactual simulations, it turns out that the differences between the right and left compensated wage elasticities are small at the intensive margin, but rather sizeable at the extensive margin. Furthermore, the compensated labor supply elasticities are larger than the uncompensated ones, but the difference is small, indicating minor income effects. The compensated and uncompensated labor supply wage elasticities vary substantially with wage rate, non-labor income, age, and number of children in two age groups. The wage elasticities are greatest for those with the lowest wages, those with the highest non-labor income, and those with many children. These results indicate that the distortionary effect of taxation is stronger for individuals with low wage rates than for those with high wage rates.

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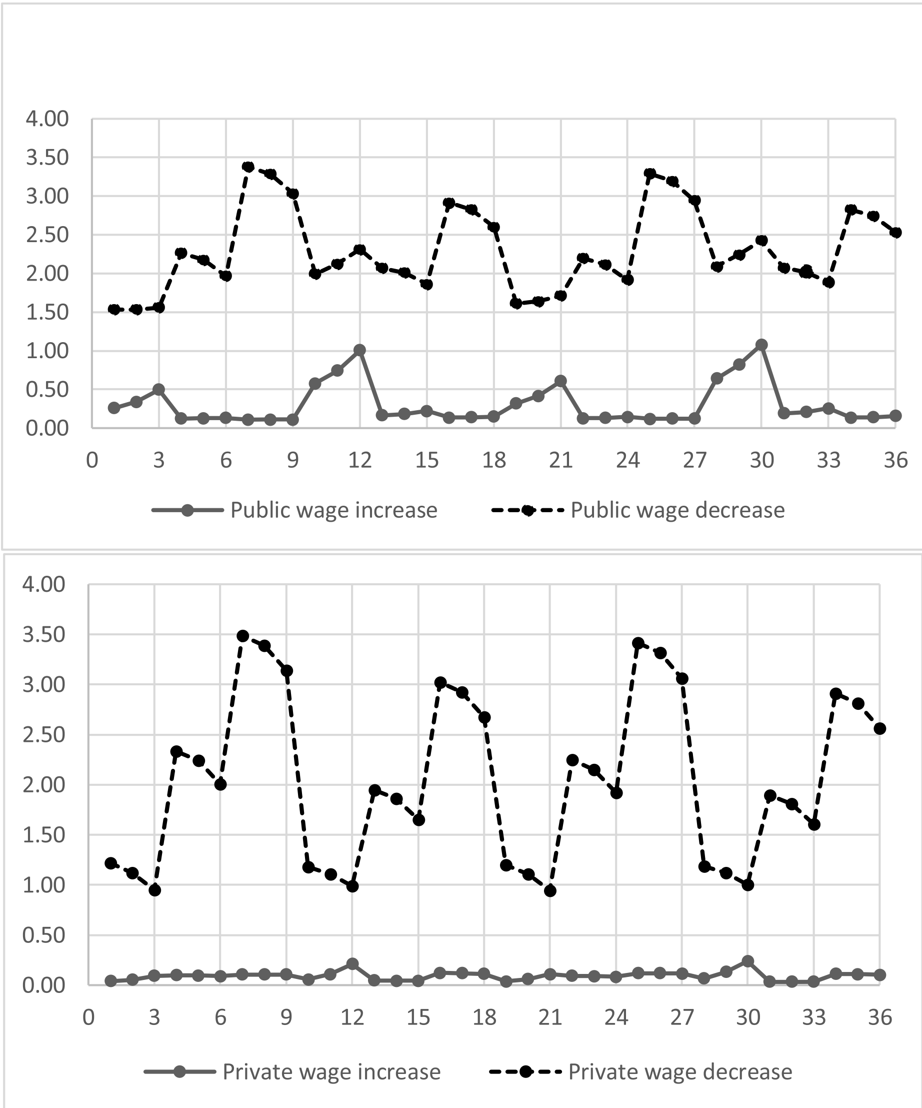

Fig. 5. Total compensated wage elasticities of unconditional expected hours.

Conflict of interest

John K. Dagsvik has received research grants from the Ragnar Frisch Centre of Economic Research, Oslo, Norway. Steinar Strøm has received research grants from the Ragnar Frisch Centre of Economic Research, Oslo, Norway. Marilena Locatelli has received research grants from the Ragnar Frisch Centre of Economic Research, Oslo, Norway. The authors declare that they have no conflict of interest.

Author statement

John K. Dagsvik has been responsible for the development of theorems and proofs Steinar Strøm has been responsible for the econometric modeling and writing Marilena Locatelli has been responsible for the software and calculations

Declaration of competing interest

John K. Dagsvik has received research grants from the Ragnar Frisch Centre of Economic Research, Oslo, Norway. Steinar Strøm has received research grants from the Ragnar Frisch Centre of Economic Research, Oslo, Norway. Marilena Locatelli has received research grants from the Ragnar Frisch Centre of Economic Research, Oslo, Norway. The authors declare that they have no conflict of interest.

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Appendix A

Marginal compensated effects for the multi-sectoral job choice model

Theorem 2. *Under Assumption 2 we have*

$$\frac{\partial^+ \log \varphi_j^c(h)}{\partial \log w_j} = \frac{w_j \partial v_j(h)}{\partial y} \sum_{x \in D \setminus \{0\}} \varphi_j(x)\left(\gamma_j(h) - \gamma_j(x)\right)_+$$

$$-w_j \sum_{x \in D \setminus \{0\}} \varphi_j(x) \frac{\partial v_j(x)}{\partial y}\left(\gamma_j(x) - \gamma_j(h)\right)_+ + w_j\left(1 - \varphi_j\right)\frac{\partial v_j(h)}{\partial y}\gamma_j(h),$$

$$\frac{\partial^- \log \varphi_j^c(h)}{\partial \log w_j} = w_j \sum_{x \in D \setminus \{0\}} \varphi_j(x) \frac{\partial v_j(x)}{\partial y}\left(\gamma_j(h) - \gamma_j(x)\right)_+ - w_j \frac{\partial v_j(h)}{\partial y} \sum_{x \in D \setminus \{0\}} \varphi_j(x)\left(\gamma_j(x) - \gamma_j(h)\right)_+$$

$$+w_j\gamma_j(h)\sum_{r \neq j, 0}\sum_{x \in D \setminus \{0\}} \frac{\partial v_r(x)}{\partial y}\varphi_r(x) + w_j\gamma_j(h)\varphi(0)\frac{\partial v(0)}{\partial y},$$

$$\frac{\partial^+ \log \varphi_j^c\left(h\right)}{\partial \log w_k} = \frac{\partial \log \varphi_j\left(h\right)}{\partial \log w_k}, \quad \frac{\partial^- \log \varphi_j^c\left(h\right)}{\partial \log w_k} = \frac{\partial \log \varphi_j\left(h\right)}{\partial \log w_k} + w_j \sum_{x \in D \setminus \{0\}}\left(\frac{\partial v_k(x)}{\partial y} - \frac{\partial v_j(h)}{\partial y}\right)\varphi_k\left(x\right)\gamma_k\left(x\right)$$

fork $\neq j$,

$$\frac{\partial^+ \log \varphi^c(0)}{\partial \log w_j} = \frac{\partial \log \varphi(0)}{\partial \log w_j}$$

and

$$\frac{\partial^- \log \varphi^c(0)}{\partial \log w_j} = \frac{\partial \log \varphi(0)}{\partial \log w_j} + w_j \sum_{x \in D \setminus \{0\}}\left(\frac{\partial v_j\left(x\right)}{\partial y} - \frac{\partial v(0)}{\partial y}\right)\gamma_j\left(x\right)\varphi_j\left(x\right)$$

where $v_j(h) = u_j(f(w_j h, y), h) + \log(\theta_j g_j(h))$ *and*

$$\gamma_j(h) = \frac{\partial v_j(h)/\partial w_j}{\partial v_j(h)/\partial y} = \frac{f_1'\left(w_j h, y\right)h}{f_2'\left(w_j h, y\right)}.$$

The Proof of Theorem 2 *is given in* Appendix B.

Corollary 5. *Under Assumption 2 it follows that*

$$\frac{\partial^+ \log \varphi_j^c\left(h\right)}{\partial \log w_j} - \frac{\partial \log \varphi_j\left(h\right)}{\partial \log w_j} = w_j \sum_{x \in D \setminus \{0\}} \varphi_j\left(x\right)\left(\frac{\partial v_j(x)}{\partial y} - \frac{\partial v_j(h)}{\partial y}\right)\min\left(\gamma_j\left(h\right), \gamma_j\left(x\right)\right)$$

and

$$\frac{\partial^- \log \varphi_j^c\left(h\right)}{\partial \log w_j} - \frac{\partial \log \varphi_j\left(h\right)}{\partial \log w_j} = w_j \sum_{x \in D \setminus \{0\}} \varphi_j\left(x\right)\left(\frac{\partial v_j(x)}{\partial y} - \frac{\partial v_j(h)}{\partial y}\right)\max\left(\gamma_j\left(h\right), \gamma_j\left(x\right)\right)$$

$$+w_j\frac{\partial v_j(h)}{\partial y}\gamma_j(h)\left(\varphi_j - 1\right) + w_j\gamma_j(h)\sum_{r \neq j, 0}\sum_{x \in D \setminus \{0\}} \frac{\partial v_r(x)}{\partial y}\varphi_r(x) + w_j\gamma_j(h)\frac{\partial v(0)}{\partial y}\varphi(0).$$

The Proof of Corollary 5 *is given in* Appendix B

First, we observe (*direct effects*) *that the marginal effect of an increase in the wage* w_j *on the probability of working* h *hours in sector* j *is similar to the marginal effect in the case of single sector model, it has a compensated term and an income effect term. As above the latter will vanish in the case of a linear utility function. The Slutsky equation in the case of a decrease in the wage rate is somewhat more complicated than* *in the one sector case.*

Second, the Slutsky equation related to an increase in wage rate w_k on the probability of working h hours in sector $j, j \neq k$, consists only of a compensated term, there is no income effect. Again, this is in accordance with the expectation based on the classical textbook case. But there is an income effect in the cross effect due to a wage decrease; again there is a wage income loss to be compensated.

Third, the marginal effect of an increase in the wage in sector j on the probability of working, consist only of a compensated term, there is no income effect. This is in accordance with the classical textbook case. However, as in the case of the single sector model, in the case of a wage decrease there is an income effect. From Theorem 2 we get

$$\frac{\partial^- \log \varphi_j^c\left(h\right)}{\partial \log w_j} - \frac{\partial^+ \varphi_j^c\left(h\right)}{\partial \log w_j} = -w_j\left(1-\varphi_j\right)\gamma_j\left(h\right)\frac{\partial v_j(h)}{\partial y} + w_j\gamma_j\left(h\right)\sum_{r\neq j}\sum_{x\in D\setminus\{0\}}\frac{\partial v_r(x)}{\partial y}\varphi_r\left(x\right) + w_j\varphi\left(0\right)\frac{\partial v(0)}{\partial y}$$

$$+w_j\sum_{r\neq j}\sum_{x\in D\setminus\{0\}}\varphi_r\left(x\right)\left(\frac{\partial v_j(x)}{\partial y} - \frac{\partial v_j(h)}{\partial y}\right)\left|\gamma_j\left(h\right) - \gamma_j\left(x\right)\right|. \tag{A.1}$$

From (A.1) we note that the sign of the difference between the right and left marginal compensating effects might happen to be both positive and negative.

Above, we have seen that the left and right marginal effects differ substantially in the sense that the algebraic expressions are quite different.

Corollary 6. *Let $\widetilde{h}_j$ and $\widetilde{h}'_j$ be independent realizations of the p.d.f. $\varphi_j(h)$. Then under Assumption 2 we have that*

$$\frac{\partial^+ E\widetilde{h}_j^c}{\partial w_j} - \frac{\partial E\widetilde{h}_j}{\partial w_j} = Cov\left(\widetilde{h}_j, -\frac{\partial v_j\left(\widetilde{h}_j\right)}{\partial y}\min\left(\gamma_j\left(\widetilde{h}_j\right), \gamma_j\left(\widetilde{h}'_j\right)\right)\right) - Cov\left(\widetilde{h}'_j, -\frac{\partial v_j\left(\widetilde{h}_j\right)}{\partial y}\min\left(\gamma_j\left(\widetilde{h}_j\right), \gamma_j\left(\widetilde{h}'_j\right)\right)\right),$$

$$\frac{\partial^- E\widetilde{h}_j^c}{\partial w_j} - \frac{\partial E\widetilde{h}_j}{\partial w_j} = Cov\left(\widetilde{h}_j, -\frac{\partial v_j\left(\widetilde{h}_j\right)}{\partial y}\max\left(\gamma_j\left(\widetilde{h}_j\right), \gamma_j\left(\widetilde{h}'_j\right)\right)\right) - Cov\left(\widetilde{h}'_j, -\frac{\partial v_j\left(\widetilde{h}_j\right)}{\partial y}\max\left(\gamma_j\left(\widetilde{h}_j\right), \gamma_j\left(\widetilde{h}'_j\right)\right)\right)$$

$$+\sum_{r\neq j}E\left(\frac{\partial v_r\left(\widetilde{h}_r\right)}{\partial y}\widetilde{h}_j\gamma_j\left(\widetilde{h}_j\right)\right) + \left(\varphi_j - 1\right)E\left(\frac{\partial v_j\left(\widetilde{h}_j\right)}{\partial y}\widetilde{h}_j\gamma_j\left(\widetilde{h}_j\right)\right) + \frac{\partial v(0)}{\partial y}\varphi\left(0\right)E\left(\widetilde{h}_j\gamma_j\left(\widetilde{h}_j\right)\right)$$

and

$$\frac{\partial^- E\widetilde{h}_j^c}{\partial w_k} - \frac{\partial E\widetilde{h}_j}{\partial w_k} = E\widetilde{h}_j E\left(\frac{\gamma_k\left(\widetilde{h}_k\right)\partial v_k\left(\widetilde{h}_k\right)}{\partial y}\right) - E\gamma_k\left(\widetilde{h}_k\right)E\left(\frac{\widetilde{h}_j\partial v_j\left(\widetilde{h}_j\right)}{\partial y}\right).$$

The Proof of Corollary 6 is given in Appendix B.

Corollary 7. *Under Assumption 2 we have that*

$$\frac{\partial^+ E\widetilde{h}_j^c}{\partial w_j} > \frac{\partial E\widetilde{h}_j}{\partial w_j}.$$

The Proof of Corollary 7 is given in Appendix B. In contrast to the single sector case we have not been able to prove that in general

$$\frac{\partial^- E\widetilde{h}_j^c}{\partial w_j} > \frac{\partial E\widetilde{h}_j}{\partial w_j}.$$

However, this inequality holds for our empirical model, cf. Section 4.2.

The formulas above concern the case where only sector specific wages are altered. It is, however, also of interest to consider the case with an overall wage increase. Specifically, assume now that $w_j = a_j w$ where w is a wage component that is common to both sectors.

Corollary 8. *Under Assumption 2 with $w_j = a_j w$ where w is a common wage component, we have for $h > 0$, that*

$$\frac{\partial^+ \log \varphi_j^c(h)}{\partial \log w} = w\frac{\partial v_j(h)}{\partial y}\sum_k\sum_{x\in D\setminus\{0\}}\varphi_k(x)\left(a_j\gamma_j(h) - a_k\gamma_k(x)\right)_+$$

$$-w\sum_k\sum_{x\in D\setminus\{0\}}\varphi_k(x)\frac{\partial v_k(x)}{\partial y}\left(a_k\gamma_k(x) - a_j\gamma_j(h)\right)_+ + \varphi_j(h)\varphi(0)a_j\gamma_j(h)\frac{\partial v_j(h)}{\partial y}$$

and

$$\frac{\partial^- \log \varphi_j^c(h)}{\partial \log w} = w\sum_k \sum_{x\in D\setminus\{0\}} \frac{\partial v_k(x)}{\partial y}\varphi_k(x)\left(a_j\gamma_j(h) - a_k\gamma_k(x)\right)_+$$

$$-w\frac{\partial v_j(h)}{\partial y}\sum_k \sum_{x\in D\setminus\{0\}} \varphi_k(x)\left(a_k\gamma_k(x) - a_j\gamma_j(h)\right)_+ + w\varphi(0)a_j\gamma_j(h)\frac{\partial v(0)}{\partial y}.$$

The Proof of Corollary 8 is similar to the Proof of Theorem 2.

Appendix B

Proofs

Consider a setting (labor market) with two levels of alternatives, sector and hours of work. Let w_j be the wage rate of sector j and y the income, $w = (w_1, w_2, ...)$, and let $\widetilde{w}$ be the corresponding vector of ex post wage rates. Let

$U_j(h, w_j, y) = v_j(h; w_j, y) + \eta_{jh}$ *and* $U(0, y) = v(0, y) + \eta_0$

be the utilities of alternative (j, h) and the non-working alternative where the error terms $\{\eta_{jh}, \eta_0\}$ are iid with c.*d.f.* $\exp(-e^{-x})$. *Let $Q^c(k, x; j, h)$ be the compensated probability of switching from hours of work x in sector k to hours of work h in sector j after a policy intervention consisting of changing w to $\widetilde{w}$, given that the ex ante and ex post utility levels are equal. The following Lemma follows from a straight forward adaptation of the result in Theorem 4 in Dagsvik and Karlström (2005).*

Lemma B1. *Under the assumptions of the multisectoral discrete labor supply model we have that*

$$Q^c(k, x; j, h) = \int_{y_{jh}}^{y_{kx}} \frac{\exp\left(v_k(x; w_k, y) + v_j\left(h; \widetilde{w}_j, z\right)\right)\partial v_j\left(h; \widetilde{w}_j, z\right)\Big/\partial z}{M(z)^2}dz \tag{B.1}$$

and

$$Q^c(j, h; j, h) = \frac{\exp\left(v_j\left(h; w_j, y\right)\right)}{M\left(y_{jh}\right)} \tag{B.2}$$

for j, k >0, x, h>0, where y_{jh} is determined by

$$v_j\left(h; w_j, y\right) = v_j\left(h; \widetilde{w}_j, y_{jh}\right) \tag{B.3}$$

and

$$M\left(z\right) = \sum_{r>0}\sum_{x\in D\setminus\{0\}} \exp(\max(v_r\left(x; w_r, y\right), v_r\left(x; \widetilde{w}_r, z\right))) + \exp\left(\max\left(v\left(0; , y\right), v\left(0; , z\right)\right)\right) \tag{B.4}$$

The formulas for $Q^c(0; j, h)$,$Q^c(k, x; 0)$ and $Q^c(0; 0)$ are obvious modifications of the formulas in (B.1) and (B.2).

Lemma B2. *Let $\Delta w_j = \widetilde{w}_j - w_j$. We have that*

$$\left(y_{jx}\left(\widetilde{w}_j\right) - y_{jh}\left(\widetilde{w}_j\right)\right)_+ = \left(\gamma_j(h) - \gamma_j(x)\right)_+\Delta w_j + o\left(\Delta w_j\right)$$

when $\Delta w_j > 0$, and

$$\left(y_{jx}\left(\widetilde{w}_j\right) - y_{jh}\left(\widetilde{w}_j\right)\right)_+ = -\left(\gamma_j(x) - \gamma_j(h)\right)_+\Delta w_j + o\left(\Delta w_j\right)$$

when $\Delta w_j < 0$, where

$$\gamma_j(x) = -y_{jx}'\left(w_j\right) =: -y_{jx}'\left(w_j\right) = \frac{\partial v_j(h)/\partial w_j}{\partial v_j(h)/\partial y} = \frac{xf_1'\left(w_jx, y\right)}{f_2'\left(w_jx.y\right)}.$$

Proof of Lemma B2

Assume first that $\Delta w_j > 0$. By the mean value Theorem we get that

$$\left(y_{jx}\left(\widetilde{w}_j\right) - y_{jh}\left(\widetilde{w}_j\right)\right)_+ = \left(\left(y'_{jx}\left(w_j\right) - y'_{jh}\left(w_j\right)\right)\Delta w_j + o\left(\Delta w_j\right)\right)_+ = \left(\left(\left(y'_{jx}\left(w_j\right) - y'_{jh}\left(w_j\right)\right)\Delta w_j\right)_+ + o\left(\Delta w_j\right)\right.$$

$$= \left(y'_{jx}(w_j) - y'_{jh}(w_j)\right)_+ \Delta w_j + o(\Delta w_j) = (\gamma_j(h) - \gamma_j(x))_+ \Delta w_j + o(\Delta w_j).$$

Assume next that $\Delta w_j < 0$. Then it follows that

$$\left(y_{jx}\left(\widetilde{w}_j\right) - y_{jh}\left(\widetilde{w}_j\right)\right)_+ = \left(\left(y'_{jx}(w_j) - y'_{jh}(w_j)\right)\Delta w_j + o(\Delta w_j)\right)_+ = \left(\left(y'_{jh}(w_j) - y'_{jx}(w_j)\right)(-\Delta w_j)\right)_+ + o(\Delta w_j)$$

$$= \left(y'_{jh}(w_j) - y'_{jx}(w_j)\right)_+ (-\Delta w_j) + o(\Delta w_j) = -(\gamma_j(x) - \gamma_j(h))_+ \Delta w_j + o(\Delta w_j).$$

Moreover, from (B.3) it follows by implicit differentiation that

$$0 = \frac{\partial v_j(x; w_j, y_{jx}(w_j))}{\partial w_j} + \frac{\partial v_j(x; w_j, y_{jx}(w_j))}{\partial y} \cdot y'_{jx}(w_j)$$

which gives

$$y'_{jx}(w_j) = -\frac{\partial v_j(x)/\partial w_j}{\partial v_j(x)/\partial y} = -\frac{f'_1(w_j x, y)x}{f'_2(w_j x, y)} = -\gamma_j(x).$$

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Proof of Theorem 2

We consider first the case of a wage increase in sector j. Let $\widetilde{w}_j = w_j + \Delta w_j$ where Δw_j is small and positive. Recall that

$$v_j(h; w_j, y) = u(f(hw_j, y), h) + \log(\theta_j g_j(h)) \text{ and } v(0; y) = u(f(0, y), 0).$$

Note that in this case $y_{kx}(w_k) = y$ for $k \neq j$ and $y_{jx}(\widetilde{w}_j) < y$. It follows from (B.1) and the mean value Theorem for integrals that when $k \neq j$,

$$\begin{aligned} Q^c(k, x; j, h) &= \frac{(\exp(v_j(x) + v_j(h))\partial v_j(h)/\partial y + O(\Delta w_j))\left(y - y_{jh}\left(\widetilde{w}_j\right)\right)}{M(y)^2} \\ &= -\varphi_j(h)\varphi_k(x) \cdot \frac{\partial v_j(h) y'_{jh}(w_j)}{\partial y} \cdot \Delta w_j + o(\Delta w_j). \end{aligned} \tag{B.5}$$

The last part of Lemma B2 yields furthermore that (B.5) can be expressed as

$$Q^c(k, x; j, h) = \varphi_j(h)\varphi_k(x)\gamma_j(h)\frac{\partial v_j(h)}{\partial y}\Delta w_j + o(\Delta w_j). \tag{B.6}$$

Similarly, we also get that

$$Q^c(0; j, h) = \varphi_j(h)\varphi(0)\gamma_j(h)\frac{\partial v_j(h)}{\partial y}\Delta w_j + o(\Delta w_j). \tag{B.7}$$

Furthermore, by using the mean value Theorem we get from (B.1) and Lemma B2 that

$$Q^c(j, x; j, h) = \varphi_j(h)\varphi_j(x)(\partial v_j(h)/\partial y)\left(y_{jx}\left(\widetilde{w}_j\right) - y_{jh}\left(\widetilde{w}_j\right)\right)_+ + o(\Delta w_j) = \varphi_j(x)\varphi_j(h)(\partial v_j(h)/\partial y)(\gamma_j(h) - \gamma_j(x))_+ \Delta w_j + o(\Delta w_j). \tag{B.8}$$

Let $\widetilde{M}(z)$ be the modification of $M(z)$ that consists in replacing w_j by $\widetilde{w}_j$. By using the mean value Theorem and (B.2) we get that

$$\begin{aligned} Q^c\left(j, h; j, h\right) - \varphi_j\left(h\right) &= \frac{\exp(v_j(h; w_j, y))}{\widetilde{M}\left(y_{jh}\right)} - \frac{\exp(v_j(h; w_j, y))}{M(y)} = \frac{\varphi_j\left(h\right)\left(M\left(y\right) - \widetilde{M}\left(y_{jh}\right)\right)}{\widetilde{M}\left(y_{jh}\right)} \\ &= -\varphi_j\left(h\right) \cdot \frac{\sum_{x \in D \setminus \{0\}}\left(\exp\left(v_j\left(x; \widetilde{w}_j, y_{jh}\right)\right) - \exp\left(v_j\left(x; w_j, y\right)\right)\right)_+}{\widetilde{M}\left(y_{jh}\right)} \end{aligned}$$

$$= -\varphi_j\left(h\right)\sum_{x\in D}\varphi_j\left(x\right)\left(\frac{\partial v_j(x)}{\partial w_j}+\frac{\partial v_j(x)}{\partial y}y_{jh}^{'}\left(w_j\right)\right)_+\Delta w_j + o\left(\Delta w_j\right) = -\varphi_j\left(h\right)\sum_{x\in D\setminus\{0\}}\varphi_j\left(x\right)\frac{\partial v_j(x)}{\partial y}\left(\gamma_j(x)-\gamma_j(h)\right)_+\Delta w_j + o\left(\Delta w_j\right) \tag{B.9}$$

Hence, it follows that

$$\frac{\partial^+\varphi_j^c\left(h\right)}{\partial w_j} = \lim_{\Delta w_j\downarrow 0}\frac{Q^c\left(0;j,h\right)+\sum_{k\neq j,0}\sum_{x\in D\setminus\{0\}}Q^c(k,x;j,h)+\sum_{x\in D\setminus\{h,0\}}Q^c(j,x;j,h)+Q^c\left(j,h;j,h\right)-\varphi_j\left(h\right)}{\Delta w_j} \tag{B.10}$$

Hence, (B.6) to (B.10) yield

$$\frac{\partial^+\log\varphi_j^c(h)}{\partial w_j} = \sum_{x\in D\setminus\{0\}}\varphi_j(x)\frac{\partial v_j(h)\left(\gamma_j(h)-\gamma_j(x)\right)_+}{\partial y} - \sum_{x\in D\{0\}}\varphi_j(x)\frac{\partial v_j(x)\left(\gamma_j(x)-\gamma_j(h)\right)_+}{\partial y}$$
$$+\sum_{k\neq j}\sum_{x\in D\setminus\{0\}}\varphi_k(x)\cdot\frac{\partial v_j(h)\gamma_j(h)}{\partial y}+\varphi(0)\frac{\partial v_j(h)\gamma_j(h)}{\partial y}$$

which also holds for $h = 0$ and therefore proves the stated result.

Consider next the cross marginal compensated effect implied by a wage rate increase in sector k, ceteris paribus. Assume that Δw_k is small and positive, $k\neq j$.Then it follows that $y_{rx} = y$ when $r\neq k$ and $y_{kx} < y$ which implies that $Q^c(r,x;j,h) = Q^c(0;j,h) = 0$ when $(r,x)\neq(j,h)$. Furthermore, it follows from (B.2) that

$$Q^c(j,h,j,h) = \frac{\exp\left(v_j\left(h;w_j,y\right)\right)}{\sum_{x\in D\{0\}}\exp\left(v\left(x;\widetilde{w}_k,y\right)\right)+\sum_{r\neq k}\sum_{x\in D\setminus\{0\}}\exp(v_r(x;w_r,y))+\exp(v(0;y))}.$$

Hence,

$$Q^c\left(j,h;j,h\right)-\varphi_j\left(h\right) = \frac{\exp\left(v_j\left(h;w_j,y\right)\right)\sum_{x\in D\setminus\{0\}}\left[\exp\left(v_k\left(x;w_k,y\right)\right)-\exp\left(v_k\left(x;\widetilde{w}_k,y\right)\right)\right]}{M(y)^2}+o\left(\Delta w_k\right)$$

By using the mean value Theorem the latter expression becomes

$$Q^c\left(j,h;j,h\right)-\varphi_j\left(h\right) = -\varphi_j\left(h\right)\sum_{x\in D\setminus\{0\}}\varphi_k(x)\frac{\partial v_k(x)}{\partial w_k}+o\left(\Delta w_k\right) = \frac{\partial\varphi_j(h)}{\partial w_k}\Delta w_k+o\left(\Delta w_k\right) \tag{B.11}$$

Hence, it follows from (B.1) and (B.11) that

$$\frac{\partial^+\varphi_j^c\left(h\right)}{\partial w_k} = \lim_{\Delta w_k\downarrow 0}\frac{\sum_r\sum_{x\in D\setminus\{0\}}Q^c\left(r,x;j,h\right)-\varphi_j\left(h\right)}{\Delta w_k} = \frac{\sum_{x\in D\setminus\{0\}}Q^c\left(j,x;j,h\right)-\varphi_j\left(h\right)}{\Delta w_k} = \frac{Q^c\left(j,h;j,h\right)-\varphi_j\left(h\right)}{\Delta w_k}.$$

Similarly, we get that $Q^c(r,x;0) = 0$ and

$$Q^c\left(0;0\right)-\varphi\left(0\right) = \frac{\partial\varphi(0)}{\partial w_k}\Delta w_k+o\left(\Delta w_k\right)$$

Thus, in this case we conclude that

$$\frac{\partial^+\varphi_j(h)}{\partial w_k} = \frac{\partial\varphi_j(h)}{\partial w_k}\text{ and }\frac{\partial^+\varphi(0)}{\partial w_k} = \frac{\partial\varphi(0)}{\partial w_k}.$$

Consider next the direct marginal effect when the wage rate in sector j decreases, that is, $\Delta w_j$is negative. In this case $y_{rx}(w_r) = y$ when $r\neq j$ and $y_{jx}(\widetilde{w}_j) > y$ for all x. As a result we obtain from (B.1) that $Q^c(r,x,j,h) = 0$ for $r\neq j$, and $Q^c(0;j,h) = 0$. In this case it follows from (B.1) and Lemma B2 that

$$Q^c\left(j,x,j,h\right) = \varphi_j\left(h\right)\varphi_j\left(x\right)\frac{\partial v_j(h)}{\partial y}\left(y_{jx}\left(w_j\right)-y_{jh}\left(w_j\right)\right)_+ + o\left(\Delta w_j\right) = -\varphi_j\left(h\right)\varphi_j\left(x\right)\frac{\partial v_j(h)}{\partial y}\left(\left(\gamma_j(x)-\gamma_j(h)\right)_+\Delta w_j+o\left(\Delta w_j\right)\right. \tag{B.12}$$

Furthermore, using (B.2) we get that

$$Q^c\left(j,h;j,h\right)-\varphi_j\left(h\right)=\frac{\exp\left(v_j(h)\right)}{\widetilde{M}\left(y_{jh}\right)}-\varphi_j\left(h\right)$$

$$=-\varphi_j\left(h\right)\frac{\sum_{x\in D\setminus\{0\}}\left(\exp\left(v_j\left(x;\widetilde{w}_j,y_{jh}\right)-\exp\left(v_j\left(x;w_j,y\right)\right)\right)\right)_+ +\sum_{r\neq j}\sum_{x\in D\setminus\{0\}}\left(\exp\left(v_r\left(x;w_r,y_{jh}\right)-\exp\left(v_r\left(x;w_r,y\right)\right)\right)\right)}{M(y)}$$

$$-\varphi_j\left(h\right)\frac{\exp\left(v\left(0,y_{jh}\right)-\exp\left(v\left(0,y\right)\right)\right)}{M(y)}+o\left(\Delta w_j\right)=-\varphi_j\left(h\right)\frac{\sum_{x\in D\setminus\{0\}}\exp\left(v_j\left(x\right)\right)\left(\frac{\partial v_j(x)}{\partial w_j}\Delta w_j+\frac{\partial v_j(x)}{\partial y}y'_{jh}\left(w_j\right)\Delta w_j\right)_+}{M(y)}$$

$$-\varphi_j\left(h\right)\frac{\sum_{r\neq j}\sum_{x\in D\setminus[0]}\exp\left(v_r\left(x\right)\right)\frac{\Delta w_j\partial v_r(x)}{\partial y}y'_{jh}\left(w_j\right)+\exp\left(v\left(0\right)\right)\frac{\Delta w_j\partial v(0)}{\partial y}y'_{jh}\left(w_j\right)}{M(y)}+o\left(\Delta w_j\right)$$

$$=\Delta w_j\varphi_j\left(h\right)\sum_{x\in D}\varphi_j\left(x\right)\frac{\partial v_j(x)}{\partial y}\left(\gamma_j(h)-\gamma_j(x)\right)_+ +\Delta w_j\sum_{r\neq j}\sum_{x\in D\setminus[0]}\varphi_r\left(x\right)\frac{\partial v_r(x)}{\partial y}\cdot\gamma_j\left(h\right)+o\left(\Delta w_j\right)+\Delta w_j\varphi_j\left(h\right)\varphi\left(0\right)\frac{\partial v(0)}{\partial y}. \tag{B.13}$$

From (B.12) and (B.13) we therefore obtain that

$$\frac{\partial^-\varphi_j^c\left(h\right)}{\partial w_j}=\lim_{\Delta w_j\uparrow 0}\frac{\sum_{x\in D\setminus\{h,0\}}Q^c(j,x;j,h)+Q^c\left(j,h;j,h\right)-\varphi_j\left(h\right)}{\Delta w_j}=-\varphi_j\left(h\right)\sum_{x\in D\setminus\{0\}}\varphi_j\left(x\right)\frac{\partial v_j(h)}{\partial y}\left(\gamma_j(x)-\gamma_j(h)\right)_+$$

$$+\varphi_j\left(h\right)\sum_{x\in D\setminus\{0\}}\varphi_j\left(x\right)\frac{\partial v_j(x)}{\partial y}\left(\gamma_j(h)-\gamma_j(x)\right)_+ +\varphi_j\left(h\right)\gamma_j\left(h\right)\sum_{r\neq j,0}\sum_{x\in D}\varphi_r\left(x\right)\frac{\partial v_r(x)}{\partial y}+\varphi_j\left(h\right)\gamma_j\left(h\right)\varphi\left(0\right)\frac{\partial v(0)}{\partial y}.$$

Consider finally the cross compensated marginal effects when $\Delta w_k<0$. In this case it follows that $y_{rx}=y$ for $r\neq k$ and $y_{kx}(\widetilde{w}_k)>y$. Hence, (B.2) implies that $Q^H(j,h;j,h)=\varphi_j(h)$. Furthermore, (B.1) implies that $Q^H(r,x;j,h)=0$ for $r\neq k$ and $(r,x)\neq(j,h)$. Hence, using (B.1) we obtain that

$$\sum_r\sum_{x\in D}Q^H(r,x;j,h)-\varphi_k(h)=\sum_{x\in D}Q^H(k,x;j,h)$$

$$=\sum_{x\in D}\left(\varphi_k\left(x\right)\varphi_j\left(h\right)\partial v_j\left(h\right)\big/\partial y+O\left(\Delta w_k\right)\right)\left(y_{kx}\left(\widetilde{w}_k\right)-y\right)$$

$$=\sum_{x\in D}\varphi_k(x)\varphi_j(h)\left(\partial v_j(h)/\partial y\right)y'_{kx}(w_k)\Delta w_k+o(\Delta w_k)$$

$$=-\varphi_j(h)\left(\partial v_j(h)/\partial y\right)\sum_{x\in D}\varphi_k(x)\gamma_k(x)\Delta w_k+o(\Delta w_k).$$

By dividing the expression above by Δw_k and letting Δw_k tend towards zero we get

$$\frac{\partial^-\varphi_j(h)}{\partial w_k}=-\varphi_j(h)\frac{\partial v_j(h)}{\partial y}\sum_{x\in D\setminus\{0\}}\varphi_k(x)\gamma_k(x).$$

Similarly, it follows that

$$\frac{\partial^-\varphi(0)}{\partial w_k}=-\varphi(0)\sum_{x\in D}\varphi_k(x)\frac{\partial v_k(x)}{\partial y}\gamma_k(x).$$

Q.E.D.

Proof of Corollary 5: It follows from Theorem 2 that

$$\frac{\partial^+\varphi_j^c\left(h\right)}{\partial w_j}-\frac{\partial\varphi_j(h)}{\partial w_j}=\frac{\partial v_j(h)}{\partial y}\varphi_j\left(h\right)\sum_{x\in D}\varphi_j\left(x\right)\left(\gamma_j(h)-\gamma_j(x)\right)_+ -\varphi_j\left(h\right)\sum_{x\in D}\varphi_j\left(x\right)\frac{\partial v_j(x)}{\partial y}\left(\gamma_j(x)-\gamma_j(h)\right)_+$$

$$-\frac{\partial v_j(h)}{\partial y}\gamma_j\left(h\right)\varphi_j\left(h\right)$$

$$+\varphi_j\left(h\right)\sum_{x\in D}\varphi_j\left(x\right)\gamma_j\left(x\right)\frac{\partial v_j(x)}{\partial y}+\varphi_j\left(h\right)\left(1-\varphi_j\right)\frac{\partial v_j(h)}{\partial y}\gamma_j\left(h\right)=-\frac{\partial v_j(h)}{\partial y}\varphi_j\left(h\right)\sum_{x\in D}\varphi_j\left(x\right)\left(\gamma_j\left(h\right)-\left(\gamma_j(h)-\gamma_j(x)\right)_+\right)$$

$$+\varphi_j\left(h\right)\sum_{x\in D}\varphi_j\left(x\right)\frac{\partial v_j(x)}{\partial y}\left(\gamma_j\left(x\right)-\left(\gamma_j(x)-\gamma_j(h)\right)_+\right)$$

Next, note that

$$\gamma_j(h) - (\gamma_j(h) - \gamma_j(x))_+ = \gamma_j(x) - (\gamma_j(x) - \gamma_j(h))_+ = \min(\gamma_j(x), \gamma_j(h)).$$

Hence, the above expression reduces to

$$\frac{\partial^+ \varphi_j^c(h)}{\partial w_j} - \frac{\partial \varphi_j(h)}{\partial w_j} = \varphi_j(h) \sum_{x \in D} \left(\frac{\partial v_j(x)}{\partial y} - \frac{\partial v_j(h)}{\partial y} \right) \varphi_j(x) \min(\gamma_j(h), \gamma_j(x)).$$

Similarly,

$$\begin{aligned}
\frac{\partial^- \varphi_j^c(h)}{\partial w_j} - \frac{\partial \varphi_j(h)}{\partial w_j} &= \varphi_j(h) \sum_{x \in D\setminus\{0\}} \frac{\partial v_j(x)}{\partial y} \varphi_j(x) (\gamma_j(h) - \gamma_j(x))_+ - \varphi_j(h) \frac{\partial v_j(h)}{\partial y} \sum_{x \in D\setminus\{0\}} \varphi_j(x) (\gamma_j(x) - \gamma_j(h))_+ - \frac{\partial v_j(h)}{\partial y} \gamma_j(h) \varphi_j(h) \\
&\quad + \varphi_j(h) \sum_{x \in D\setminus\{0\}} \varphi_j(x) \gamma_j(x) \frac{\partial v_j(x)}{\partial y} + \varphi_j(h) \gamma_j(h) \sum_{r \neq j} \sum_{x \in D\setminus\{0\}} \frac{\partial v_r(x)}{\partial y} \varphi_r(x) + \varphi_j(h) \gamma_j(h) \varphi(0) \frac{\partial v(0)}{\partial y} \\
&= \varphi_j(h) \sum_{x \in D\setminus\{0\}} \varphi_j(x) \frac{\partial v_j(x)}{\partial y} \left((\gamma_j(h) - \gamma_j(x))_+ + \gamma_j(x) \right) - \varphi_j(h) \frac{\partial v_j(h)}{\partial y} \sum_{x \in D\setminus\{0\}} \varphi_j(x) \left((\gamma_j(x) - \gamma_j(h))_+ + \gamma_j(h) \right) \\
&\quad + \frac{\partial v_j(h)}{\partial y} \gamma_j(h) \varphi_j(h) (\varphi_j - 1) + \varphi_j(h) \gamma_j(h) \sum_{r \neq j} \sum_{x \in D\setminus\{0\}} \frac{\partial v_r(x)}{\partial y} \varphi_r(x) + \varphi_j(h) \gamma_j(h) \varphi(0) \frac{\partial v(0)}{\partial y} \\
&= \varphi_j(h) \sum_{x \in D\setminus\{0\}} \varphi_j(x) \left(\frac{\partial v_j(x)}{\partial y} - \frac{\partial v_j(h)}{\partial y} \right) \max(\gamma_j(x), \gamma_j(h)) + \gamma_j(h) \varphi_j(h) \left(\sum_{r \neq j} \sum_{x \in D\setminus\{0\}} \frac{\partial v_r(x)}{\partial y} \varphi_r(x) \right) + \varphi_j(h) \gamma_j(h) \varphi(0) \frac{\partial v(0)}{\partial y}.
\end{aligned}$$

. Q.E.D.

Proof of Corollary 6: From Theorem 2 it follows that

$$\begin{aligned}
\sum_h \left(\frac{\partial^+ h\varphi_j^c(h)}{\partial w_j} - \frac{\partial h\varphi_j(h)}{\partial w_j} \right) &= \sum_h \sum_x h\varphi_j(h) \frac{\partial v_j(x)}{\partial y} \min(\gamma_j(h), \gamma_j(x)) \varphi_j(x) - \sum_h \sum_x h\varphi_j(h) \frac{\partial v_j(h)}{\partial y} \min(\gamma_j(h), \gamma_j(x)) \varphi_j(x) \\
&= -E\left(-\frac{\partial v_j(\tilde{h}'_j)}{\partial y} \tilde{h}_j \min\left(\gamma_j(\tilde{h}_j), \gamma_j(h'_j)\right) \right) + E\left(-\frac{\partial v_j(\tilde{h}_j)}{\partial y} \tilde{h}_j \min\left(\gamma_j(\tilde{h}_j), \gamma_j(h'_j)\right) \right) \\
&= Cov\left(\tilde{h}_j, -\frac{\partial v_j(\tilde{h}'_j)}{\partial y} \min\left(\gamma_j(\tilde{h}_j), \gamma_j(h'_j)\right) \right) - Cov\left(\tilde{h}_j, -\frac{\partial v_j(\tilde{h}_j)}{\partial y} \min\left(\gamma_j(\tilde{h}_j), \gamma_j(h'_j)\right) \right)
\end{aligned}$$

The remaining part of Corollary 6 is proved in a similar way.
Q.E.D.
Proof of Corollary 7: Recall that $-\partial v_j(h)/\partial y$ *is increasing in h. The covariance*

$$Cov\left(\tilde{h}_j, -\frac{\partial v_j(\tilde{h}_j)}{\partial y} \min\left(\gamma_j(\tilde{h}_j), \gamma_j(h'_j)\right) \right)$$

is therefore positive and greater than the covariance

$$Cov\left(\tilde{h}_j, -\frac{\partial v_j(\tilde{h}'_j)}{\partial y} \min\left(\gamma_j(\tilde{h}_j), \gamma_j(h'_j)\right) \right)$$

because in the latter expression $\partial v_j(\tilde{h}'_j)/\partial y$ *is independent of* $\tilde{h}_j$*. Hence, the result of Corollary 7 follows from Corollary 6.*

Q.E.D.

Appendix C. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jocm.2021.100326.

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