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A European equivalence scale for public in-kind transfers

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Abstract
This paper introduces a theory-based equivalence scale for public in-kind transfers, which justifies comparison of distributions of extended income (cash income plus the value of public services) between European countries. We demonstrate the usefulness of the proposed equivalence scale in an empirical analysis of the effects of public health care, long-term care, education and childcare expenditure on estimates of income inequality and poverty for 24 European countries. The empirical results show significant effects of public in-kind transfers on the level of income inequality and poverty for all countries. Over the period 2006–2018, inequality and poverty estimates display rather different trends across European countries.

Keywords Income distribution · Poverty · Equivalence scales · Relative needs · Public services · Public in-kind transfers

JEL Classification D30 · H40 · I30

1 Introduction

The increase in economic inequality in European countries during the past four decades has received much attention; not least due to Piketty's (2014) and Scheidel's (2017) dismal predictions of a future class-divided "Downton Abbey" society. This prediction has been justified by historic data records as well as the broadly documented recent rise in wealth and market income inequality. The exclusive focus on the evolution of the distribution of individual pre-tax market income has been dictated by historic data limitations. However, distributions of individual market incomes, even when taxes are subtracted, do not provide

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a complete picture of present distributions of economic well-being in European countries, since publicly funded welfare services constitute a substantial part of the welfare states' transfers to individuals and households (OECD 2017). While market income could be considered as an appropriate measure of economic well-being before and shortly after the Second World War, this is clearly not the case for the past four decades. Moreover, there is large variation in the spending on public in-kind transfers across European countries. Northern European countries as well as France, Germany, Belgium and the Netherlands spend a relatively large share of GDP on public welfare services, whereas Southern and Eastern European countries spend much less on public services. However, even the countries with the lowest share spend almost 8 % of GDP on public in-kind transfers. Thus, the omission of public in-kind transfers from a measure of economic well-being may call into question the validity of income comparisons over time for a given country, and across countries with different levels and composition of taxes, cash benefits and publicly provided goods and services. In taking notice of this fact, several researchers have acknowledged the importance of incorporating public in-kind transfers in studies of inequality (Atkinson et al. 2002; Stiglitz et al. 2009; Congressional Budget Office 2011; OECD 2011). This shift in focus calls for broader measures of household resources that reflect a comprehensive view of how government redistribution affects household living standards.[1]

The importance of accounting for economies of scale in consumption in analyses of distributions of disposable cash income is universally acknowledged. Equivalence scales designed to account for the consumption needs associated with cash income might however be inappropriate when analysing measures of income that include the value of public services, since these scales ignore the presence of free or subsidised public services such as education and health care. For instance, elderly people have higher needs for health care since they in general suffer from poorer health status, whereas children are considered to have high needs for education services. Since such services mainly are received by children and the elderly, it is not plausible to assume that the needs for public services are proportional to the needs for cash income. Consequently, extended income studies relying on equivalence scales designed for cash income might overstate the affluence among the elderly and families with children, since they fail to account for the relatively high needs of such families for public in-kind transfers. Thus, it is required to introduce a separate equivalence scale for public in-kind transfers and to combine it with the EU-scale for cash income to obtain an equivalence scale for extended income.

Pollak and Wales (1979) show that any attempt to recover equivalence scales from household behaviour encounters the problem that household choices are conditional on needs, which makes it inherently difficult to disentangle needs from preferences. As a result, estimation of equivalence scales from demand data requires identifying assumptions that are untestable (Blundell and Lewbel 1991). Browning et al. (2013) and Chiappori (2016) suggest an empirical approach that makes situational comparisons of the cost of putting the individual on the same indifference curve in different family contexts. However, given that household demand and market prices for public in-kind transfers are not observed, such demand-based approaches are ruled out from estimation of equivalence scales that are designed to capture the specific needs associated with public services.

An alternative approach is to base the judgment of the relative needs of recipients on the imputation of public values attributed by the government. Despite the difficulties involved in making interpersonal welfare comparisons, government authorities are forced to make

[1] For recent studies of the impact of in-kind benefits on the income distribution, see e.g. Figari and Paulus (2015), Aaberge et al. (2010a, b, 2017, 2018, 2019, 2021) and Piketty et al. (2017).

such comparisons in decision processes by which public in-kind transfers are allocated across individuals and households. Yet, there is little prior work that employs the targeting of public in-kind transfers as a source of information on the equivalence scales that are implicit in the transfer system. A notable exception is Olken (2005), who proposes a method for identifying "community equivalence scales" from a subsidised rice program to poor households that is allocated by local authorities in Indonesia. Similarly, Aaberge et al. (2010a, b, 2019) estimate the equivalence scales implicitly used in local government allocations of public in-kind transfers in Norway.

A major purpose of the present paper is to introduce a common European equivalence scale for extended income by including most of the government spending on public in-kind transfers. To this end, we introduce a theory-based equivalence scale for public in-kind transfers that can be used as a basis for comparing distributions of extended income across countries. The proposed equivalence scale is derived from a social welfare function which is shown to satisfy a generalised multiple goods' version of the Pigou-Dalton principle of transfers. An advantage of our approach is that measures of equivalence scales, welfare, inequality and poverty constitute a coherent framework that ensures internal consistency between the different methodological parts and moreover has a transparent normative justification.

This paper contributes to the literature on equivalence scales by introducing a scale for public in-kind transfers that is shown to satisfy extended versions of well-known normative principles and axioms. The proposed scale has similar convenient properties as the EU-scale for cash income and is thus easy to apply when data for the household composition by age are available. Both scales consider needs in a relative sense by defining needs as the ratio between income levels (or expenditure), which yield the same level of well-being for two different household types. Alternatively, needs might be defined as the difference in income between two different household types which is required to attain the same level of well-being. While the former is denoted a relative equivalence scale, the latter is denoted an absolute equivalence scale. To clarify the difference between the relative and absolute scales, we have included a methodological discussion that contrasts the equivalence scale approach proposed in this paper to an absolute scale employed for applied work. Our approach assumes that the mean public in-kind transfers received by different target groups reflect the value judgments regarding *relative* needs across recipent types. Thus, for identification we rely on the implicit value judgements that are revealed by the in-kind transfer system. Since such value judgments may, as for cash income, differ across nations, we employ a cross-country average of the estimated equivalence scales to obtain a measure of the average value judgments in European countries regarding relative needs for in-kind transfers.

To provide information on the effect of public in-kind transfers on poverty and income inequality, we include an empirical application for 24 European countries based on a broader measure of economic well-being than cash income by accounting for the values of basic public services such as childcare, education, health care and long-term care. The services are targeted to individuals who belong to well-defined subgroups of the population and amount to a sizeable share of public spending in all countries. Allocation of public expenditure to different services is reported by Eurostat in the Education Database and the System of Health Accounts, while we also utilise spending profiles by age and gender reported by the European Commission.

Households' cash incomes and demographic and household data have been made available by the European Union Statistics on Income and Living Conditions (EU-SILC). The measure of of disposable cash income used in this study is consistent with the measure

used by official statistics, Eurostat and OECD, and does not account for indirect taxes such as value added taxes and other consumption taxes. While interesting and relevant, imputation of indirect taxes requires microdata on consumption expenditure that are not available in EU-SILC. Studies that account for indirect taxes find however that such taxes tend to be regressive in the sense that average tax burden decreases with income rank (Garfinkel et al. 2006; Decoster et al. 2010; Figari and Paulus 2015). Hence, accounting for indirect taxes will be expected to increase estimates of inequality and poverty.

The empirical part of the paper includes new results that show time trends from 2006 to 2018 in estimates of inequality and poverty, by country and for different income definitions. Decompositions of inequality and poverty measures by income sources and population subgroups by age is an integral part of the empirical analysis. Moreover, we evaluate the sensitivity of inquality and poverty estimates both regarding the choice of equivalence scale and measure of inequality, and with regard to inefficiency in the production of public services.

The paper is organised as follows. Section 2 introduces a theoretical justified needs-adjusted (NA) equivalence scale that forms the basis for deriving a common equivalence scale for European countries. Section 3 discusses data, empirical implementation and methods used for analysing income inequality and poverty in 24 European countries. Section 4 displays the results of the empirical analysis. Section 5 concludes.

2 Needs for public in-kind transfers and equivalence scales

Equivalence scales are used to justify comparisons of disposable incomes of households who differ in needs. The relative equivalence scale for a given household shows the scale rate of income that a specific household needs to obtain the same level of well-being as the reference household. When disposable cash income is used as a measure of economic well-being, the common practice is to employ the EU scale as a means for achieving interpersonal comparability in analyses of inequality and poverty.[2] While theoretically justified equivalence scales can be constructed from household expenditure functions, most empirical analyses typically use more pragmatic scales to adjust incomes for differences in household size and composition (see e.g. Coulter et al. 1992). However, since the commonly used equivalence scales are designed to account for differences in household's relative needs for disposable cash income, they are not necessarily appropriate when analysing a measure of economic welfare that also includes the value of public in-kind transfers. For example, the conventional scales do not acknowledge that the needs for health services and education are relatively high among the elderly and households with children. Consequently, the economic well-being of households with relatively high needs for public in-kind transfers might be overrated by studies that apply the conventional equivalence scales as conversion factors for measures of income that include the value of public in-kind transfers.

A major objective of this paper is to introduce an equivalence scale that relaxes the assumption that the relative needs of different household types are unaffected when the definition of income is extended to account for the value of public in-kind transfers. To

[2] The EU scale is also called the modified OECD scale in the literature. This scale assigns weight 1 to the first adult of the household, 0.5 to each additional member aged 14 and above and 0.3 to children aged under 14. Economies of scale in consumption is the rationale for assigning a higher weight to the household head. Cars and housing are examples of jointly consumed goods, which are assumed to contribute to economies of scale.

account for heterogeneity in relative needs for cash income, we rely on the conventional EU scale. As is well known, the EU scale assigns relatively low weight to children, simply because children have smaller needs for private consumption than adults. However, when relative needs are also supposed to incorporate public education services, it follows that the equivalence scale factor for children will become larger. Moreover, the equivalence scale should also account for differences in relative needs between adults of different ages when the income definition includes public health care and long-term care.

2.1 Relative versus absolute equivalence scales

The purpose of an equivalence scale is to convert household incomes into comparable individual-specific incomes (equivalent incomes). Equivalence scales might be absolute or relative. A relative scale provides the rate at which one Euro for one household translates into the Euro amount that will produce the same well-being for another household. Thus, if household h enjoys income y_h, and m_h is the conversion rate from the reference household to household h, then the reference household needs income y_h/m_h to obtain the same level of well-being as (members of) household h enjoys. Thus, y_h/m_h is denoted the equivalent income of household h, and it follows that the relative scale is given by the ratio of income to equivalent income. By contrast, an absolute equivalence scale is given by the difference between income and equivalent income, which means that the reference household needs income $y_h - c_h$ to attain the same level of well-being as household h, where c_h is the absolute scale or additive conversion factor from the reference household to household h.

An equivalence scale is said to be exact if it does not depend on income (Lewbel 1989; Blackorby and Donaldson 1993). Exact relative scales are the ones commonly used. In line with standard practice, our aim is to develop an exact relative equivalence scale for extended income that is based on a transparent normative justification. Our approach assumes that the relative need for public services constitutes a share of extended income that is depending on household type, but not on the level of income.

Paulus et al. (2010) introduced an alternative equivalence scale by relying on a "fixed cost" approach, which assumes that household needs for public services equals a fixed amount of money that depend on household type but not on income. However, when the fixed cost approach for non-cash income is combined with an exact relative equivalence scale for cash income (e.g. the EU scale), the resulting combined scale for extended income will depend on the income level of the households. To illustrate the difference between combining a relative and an absolute equivalence scale and two relative equivalence scales we will use the following example:

Example Table 1 displays three different comparable extended incomes for a single adult and a single parent. In each of the cases, we assume that the two households attain equal level of well-being (equivalent extended income) when we apply an exact relative equivalence scale and/or an income-dependent scale of the type proposed by Paulus et al. (2010). In Case 1, we consider a single adult with income 100 and a single parent with one child, and with income 130. Since the scale factors for the single adult and the single parent according to the EU scale are 1 and 1.3, the cash incomes of these two households produce the same well-being. Furthermore, assume that non-cash income 10 is added to the cash income of the single adult and 55 for the non-cash income of the single parent. By treating 10 and 55 as absolute needs levels for non-cash income, the conversion rate of the combined scale for extended income proposed by Paulus et al. (2010) becomes,

Table 1 The impact of choice of equivalence scale for three different cases

	Case 1		Case 2		Case 3	
Family type	Single adult	Single parent	Single adult	Single parent	Single adult	Single parent
Cash income	100	130	200	260	200	260
Non-cash income	10	55	20	110	20	70
Extended income	110	185	220	370	220	330

Note: The calculations required to produce Table 1 is explained in the Appendix

as demonstrated in the Appendix, equal to the conversion rate of the exact relative scale (185/110) = 37/22. Thus, in Case 1 the two scales agree that the single adult and the single parent enjoy the same well-being level.

Next, assume that the cash and non-cash income of the single adult are doubled, whereas only the cash income of the single parent is doubled. Then the following question arises: how much non-cash income would the single parent household need to become equally well off as the single adult? The details of the following calculations are provided in the Appendix: Using the exact relative equivalence scale we demonstrate that the single parent needs 110 in non-cash income in this case, which means that Case 2 preserves equality in well-being between the two households. By contrast, the income-dependent equivalence scale proposed by Paulus et al. (2010) requires 70 in non-cash income to make the well-being levels of the two households equal (Case 3 of Table 1), which is 36% less than what was required when the relative equivalence scale was used.

Table 1 illustrates how the exact relative equivalence scale differs from the "fixed cost" measure of needs for public services. Given that the two households are equally well off in Case 1, the disagreement concerns whether it is Case 2 or Case 3 that preserves equality. Paulus et al. (2010) argue that public services such as education and health care are necessary goods with recipient needs that are little affected by income. However, if this is the case then it might be difficult to explain why richer countries mostly spend a larger share of their national income on public services than poorer countries (see Fig. 1 below). Moreover, in countries where education and health care are private market goods, richer households demand considerably more extensive services than poorer households. If public services were necessary goods, we would expect to observe budget shares that decline with total income (Engel's law), rather than budget shares that increase with total income (Wagner's law).

The combination of a relative scale for cash income and an absolute scale for non-cash income also raises the question of whether public cash transfers should be treated similarly as public services like education and health care. Moreover, since the use of relative and absolute equivalence scales have very different implications, it is required to provide a normative justification for combining a relative scale for cash income with an absolute scale for public in-kind transfers. We do however not know of any normative justification for this approach in the literature. Moreover, we question whether it makes sense to use an absolute scale for public in-kind transfers when relative measures of income inequality (Gini coefficient) and poverty form the basis of the anlysis; not least since Ebert and Moyes (2003) have demonstrated that a convincing normative justification of using relative measures of inequality requires use of relative equivalence scales.

By considering primary and secondary education as an absolute need, Callan and Keane (2009) claim that these in-kind transfers do not contribute to well-being and thus do not affect income inequality. A concern with their "cancelling out" approach is that governments in different countries provide different amounts of public services and moreover

have increased spending on public services over time. This could either mean that absolute need varies significantly across countries and over time, or that there are differences in the generosity of public service provision in the sense that the extent of public in-kind transfers varies relative to needs. For consistency, the assumption of absolute needs for public services makes it also necessary to introduce absolute needs for private consumption goods, which suggests that conventional studies based on country-specific distributions of disposable cash income suffer from lack of comparability. By contrast, we will instead interpret the significant cross-country variation in the extent of public service provision across European countries as an indicator of differences in material well-being.

The choice of the EU scale (modified OECD scale) as the baseline equivalence scale for cash income is motivated by common practice in empirical research. This scale was originally proposed by Hagenaars et al. (1994), who relied on both theoretical and empirical evidence. Moreover, recent studies by Van de Ven et al. (2017) and Dudel et al. (2020) provide empirical evidence in favor of the EU-scale. Like the approach used for estimating the equivalence scale for public in-kind transfers in this study, Van de Ven et al. (2017) use country-specific averages that are implicit in the tax-transfer systems of European countries as a basis for estimating the equivalence scale for disposable cash income.

Using a common European equivalence scale makes cross-country comparisons more transparent than comparison based on different country-specific equivalence scales. In the latter case ranking of countries by income inequality might depend equally much on differences in scales as on differences in distributions of income, which will make it hard to interpret the ranking results. Thus, rather than relying on country-specific equivalence scales, we will derive a relative equivalence scale that is common for European countries. The common equivalence scale will together with the Gini coefficient and the Head Count measure of poverty constitute a coherent methodological framework that ensures a comparable analysis of inequality and poverty across European countries.

2.2 Social welfare and needs-adjusted public in-kind transfers

This subsection describes the social evaluation framework that is used to derive a common European equivalence scale for public in-kind transfers. To this end, it is required to introduce the following notation: Consider a country k with H_k households and let x_{0hk} denote the cash income of household h that is disposable for consumption of market goods. Let $(x_{1hk}, x_{2hk}, \ldots, x_{Shk})$ denote the values of S public services received by household h, $\mathbf{x_{hk}} = (x_{0hk}, x_{1hk}, \ldots, x_{Shk})$, $h = 1, 2, \ldots, H_k$; $k = 1, 2, \ldots, K$, where $\mathbf{x_{hk}}$ is a vector of cash income (composite consumption) and public in-kind transfers for H_k households in country k. To account for heterogeneity in relative needs of goods and services in comparisons of $\mathbf{x_{hk}}$, $h = 1, 2, \ldots, H_k$; $k = 1, 2, \ldots, K$, we introduce the vector $\boldsymbol{\gamma}_{\mathbf{hk}} = (\gamma_{0hk}, \gamma_{1hk}, \ldots, \gamma_{Shk})$ of good-specific needs parameters, which may differ across households as well as across countries. The needs parameters are assumed to form an integral part of the following much used CES-family of measures of well-being[3]

[3] Blackorby and Donaldson (1982) provide an axiomatic justification of the well-being measure defined by (1). As Decancq and Lugo (2012, 2013) and Seth (2013), we assume that the well-being measure is homothetic, which means that the social evaluation is not affected by scale transformations of income.

$$W_{hk} = \left[\sum_{i=0}^{S} \frac{\gamma_{ihk}}{\gamma_{+hk}} \left(\frac{x_{ihk}}{\gamma_{ihk}} \right)^{1-\tau} \right]^{\frac{1}{1-\tau}}, h = 1, 2, \ldots, H_k, k = 1, 2, \ldots, K \tag{1}$$

where ${}_{S}W_{hk}$ is the well-being obtained by household h living in country k, and $\gamma_{+hk} = \sum_{i=0}^{S} \gamma_{ihk}$, $\gamma_{++k} = \sum_{i=0}^{} \sum_{h=1}^{H_k} \gamma_{ihk}$. The well-being index is defined as a weighted mean of order $1-\tau$ of the normalised goods (x_{ihk}/γ_{ihk}). The shape of the iso-well-being curves is determined by the commodity-specific weights $(\gamma_{ihk}/\gamma_{+hk})$ and by the degree of complementarity between goods (τ). The case with $\tau = 0$ corresponds to perfect substitutes, whereas $\tau \to \infty$ corresponds to perfect complements.

When the γ_{ihk}-parameters depend on the size and the demographic composition of households, the well-being measure defined by (1) belongs to the family that satisfies "demographic scaling" (Pollak and Wales 1981), which was first proposed by Barten (1964).[4] Since the need parameter γ_{ihk} is included in the welfare function as a scaling parameter, it can be interpreted as the number of "equivalent adults" associated with good i. The rescaling of goods by needs parameters means that the normalised goods are all measured in units of need. Moreover, the commodity-specific weights are chosen to account for different relative needs for different goods of different households.[5]

Next, social welfare is aggregated across households within a country by employing the following additive separable function:

$$W_k = \frac{1}{\gamma_{++k}} \sum_{h=1}^{H_k} \gamma_{+hk} V\left(W_{hk}\right), \tag{2}$$

where $V(W_{hk})$ is the contribution to total welfare from the well-being of household h in country k. Note that the assumption of a common welfare function V for all households justifies comparisons of the welfare levels $V(W_{hk})$ across households in country k. The welfare weights of the social welfare function are assumed to be proportional to the total needs of each household (γ_{+hk}).[6] This means that households with equal needs are treated symmetrically and moreover that a higher welfare weight is assigned to a household with higher needs than to a household with lower needs.

The common welfare function V is assumed to increase in household well-being, which implies that social welfare increases if a household gets a partial increase in the consumption of any market good or any in-kind transfer. For households with identical needs parameters, satisfaction of Pigou-Dalton's principle of transfers requires that V is concave. Moreover, as demonstrated by Ebert (1995, 1997, 1999) and Ebert and Moyes (2003) the principle of transfers for households with identical needs parameters can be generalised to the case with heterogeneous households by invoking a principle of between type transfers (BTT). When households differ in needs-related characteristics, a transfer of income from a richer to a poorer household will not necessarily imply a reduction in well-being inequality

[4] This means that the impact of family size and composition on well-being is treated as analogous to a price distortion. Chiappori (2016) provides a theoretical justification of commodity-specific scaling of the Barten type.

[5] Hence, needs parameters provide sufficient information to identify both the commodity-specific weights and the normalising transformation. Note, however, that by allowing the needs parameters to differ across households, the specification (1) is more flexible than conventional well-being indices which ignore heterogeneity in needs.

[6] If government decision rules are affected by other factors besides pure welfare maximisation, the welfare weights and equivalence scales that are implicit in government targeting reflect the combined impact of needs assessment and those other factors on the allocations (Olken 2005).

(see Glewwe 1991). This paradox, which arises because the interaction between needs and income might offset the effect of differences in incomes on differences in well-being, was used by Ebert (1995) as a justification for introducing a transfer principle defined in terms of needs-adjusted incomes. In the case with one composite good, Ebert (1997) proved that the welfare function defined by (1) and (2) for increasing concave V satisfies the BTT principle, which means that the BTT principle provides a normative justification for employing needs dependent welfare weights in analyses of the distribution of income for individuals living in heterogeneous households.

As a reference point for evaluating distributions of extended income, the ethical observer is assumed to rely on the optimal allocation of household consumption according to (1).[7] Conditions of allocative efficiency allow us to treat public in-kind transfers as non-distorting and fungible with cash income. A concern with this assumption is that there are quantity constraints associated with public provision of public in-kind transfers, which might lead to efficiency losses in intra-household allocations. However, the effect of this source of inefficiency may diminish if households have access to private substitutes for public in-kind transfers. Indeed, the evidence presented in Cunha (2014), Fraker et al. (1995), Hoynes and Schanzenbach (2009), Moffitt (1989) and Slesnick (1996) suggests relatively small consumption distortions from public in-kind transfers.

As indicated above we will obtain a measure of well-being by maximising W_{hk} defined by (1) with respect to consumption of market goods and public in-kind transfers, and subject to the budget constraint

$$C_{hk} \equiv x_{+hk} = \sum_{i=0}^{S} x_{ihk} \tag{3}$$

which yields the first order conditions

$$\frac{x_{ihk}}{\gamma_{ihk}} = \frac{x_{+hk}}{\gamma_{+hk}} = \frac{C_{hk}}{\gamma_{+hk}}, i = 0, 1, \ldots, S. \tag{4}$$

When (2.4) is satisfied for all households, then $W_{hk} = C_{hk}/\gamma_{+hk}$ for all h, which means that C_{hk}/γ_{+hk} can be considered as an inter-household comparable measure of material living standard. A scale transformation of C_{hk}/γ_{+hk} will be called equivalent income (or needs-adjusted income) below. If any two households obtain equal needs-adjusted incomes it follows that they also obtain equal well-being.

To extend the single good BTT principle to the case of multiple goods it is convenient to introduce a definition of *needs-adjusted progressive transfers*.

Definition I Let $E(C, \gamma)$ denote the equivalent income of a household with extended income C and needs parameter γ. Then the distribution $\left(\widetilde{C}_{1k}, \widetilde{C}_{2k}, \ldots, \widetilde{C}_{H_k k}\right)$ is said to be obtained from the distribution $\left(C_{1k}, C_{2k}, \ldots, C_{H_k k}\right)$ by *a needs-adjusted progressive transfer* $\delta > 0$ if for households g and j in country k the following conditions are satisfied

[7] It is not necessary to assume that households are consuming optimal vectors of goods, only that the ethical observer is using optimal vectors as a benchmark for comparing the welfare of different households. Alternatively, when $\tau = 0$, public in-kind transfers and cash incomes are treated as perfect substitutes by the ethical observer.

$$\widetilde{C}_{gk} = C_{gk} + \delta \text{ and } \widetilde{C}_{jk} = C_{jk} - \delta,$$
$$\widetilde{C}_{hk} = C_{hk}, \text{ for all } h \neq g, j, \text{(and)}$$
$$E\left(C_{gk}, \gamma_{+gk}\right) < E\left(\widetilde{C}_{gk}, \gamma_{+gk}\right) \leq E\left(\widetilde{C}_{jk}, \gamma_{+jk}\right) < E\left(C_{jk}, \gamma_{+jk}\right)$$

Definition I, which can be considered as an extension of the definition of progressive transfers to the case with heterogeneous households, forms a useful basis for a multiple good extension of the BTT principle.

Definition II (*The principle of between type transfers of multiple goods* (*BTT*)). Let $\left(\mathbf{x_{1k}}\left(C_{1k}, \gamma_{+1k}\right), \ldots, \mathbf{x_{H_k k}}\left(C_{H_k k}, \gamma_{+H_k k}\right)\right)$ be the vector of goods that maximises the welfare of household h in country k for given extended income C_{hk}, $h = 1, 2, \ldots, H_k$, when welfare is evaluated according to the social welfare function $W_k\left(\mathbf{x_{1k}}, \mathbf{x_{2k}}, \ldots, \mathbf{x_{H_k k}}\right)$. The welfare function W_k is said to satisfy the principle of between type transfers if

$$W_k\left(\mathbf{x_{1k}}\left(\widetilde{C}_{1k}, \gamma_{+1k}\right), \ldots, \mathbf{x_{H_k k}}\left(\widetilde{C}_{H_k k}, \gamma_{+H_k k}\right)\right) > W_k\left(\mathbf{x_{1k}}\left(C_{1k}, \gamma_{+1k}\right), \ldots, \mathbf{x_{H_k k}}\left(C_{H_k k}, \gamma_{+H_k k}\right)\right)$$

when $\left(\widetilde{C}_{1k}, \widetilde{C}_{2k}, \ldots, \widetilde{C}_{H_k k}\right)$ is obtained from $\left(C_{1k}, C_{2k}, \ldots, C_{H_k k}\right)$ by means of a sequence of needs-adjusted progressive transfers.

The following proposition shows that the principle of BTT imposes the condition of concave V on the social welfare functions defined by (1) and (2).

Proposition I The social welfare functions defined by (1) and (2) satisfy the principle of BTT for strictly increasing concave V.

The proof is given in Appendix I.

2.3 Needs-adjusted equivalence scales

Inserting (4) in (1) yields

$$W^*_{hk}\left(C_{hk}\right) = \frac{C_{hk}}{\gamma_{+hk}}, h = 1, 2, \ldots, H_k, k = 1, 2, \ldots, K, \tag{5}$$

where W^*_{hk} is the maximum welfare that can be obtained for household h for given extended income C_{hk}. Furthermore, the solution of the dual problem of minimising the cost to obtain the welfare level W_{hk} for household h is given by the cost function

$$C^*_{hk}\left(W_{hk}\right) = W_{hk}\gamma_{+hk}, h = 1, 2, \ldots, H_k, k = 1, 2, \ldots, K. \tag{6}$$

By employing the cost functions (6), we get the following family of relative equivalence scales:

$$NA_{hk} = \frac{C^*_{hk}\left(W_{rk}\right)}{C^*_{rk}\left(W_{rk}\right)} = \frac{\gamma_{+hk}}{\gamma_{+rk}}, h = 1, 2, \ldots, H \tag{7}$$

where NA_{hk} is the scale factor for household h and $C^*_{rk}(\cdot)$ is the cost function of the reference household r in country k. Equivalent income is defined by $E_{hk} = C_{hk}/NA_{hk}$, which

represents a money measure of material living standard, and is interpreted as the minimum cost required for the reference household to attain the same welfare level as household h enjoys from extended income C_{hk}.

The equivalence scale NA_{hk} is called a needs-adjusted scale since it can be considered as an ordinary scale for cash income adjusted for the needs of services provided by local and central governments. This is demonstrated by the following decomposition of (7),

$$NA_{hk} = \theta_{rk} CI_{hk} + \left(1 - \theta_{rk}\right) NC_{hk} \tag{8}$$

where $CI_{hk} = \gamma_{0hk}/\gamma_{0rk}$ is the equivalence scale for cash income, $NC_{hk} = (\gamma_{+hk} - \gamma_{0hk})/(\gamma_{+rk} - \gamma_{0rk})$ is the non-cash scale for the value of public service provision, and $\theta_{rk} = \gamma_{0rk}/\gamma_{+rk}$ is the weight assigned to cash income in the composite NA scale for extended income. The weight θ_{rk} is equal to the share attributed to cash income in the needs for extended income of the reference household r.

An equivalence scale is said to be exact if it does not depend on the income level. The commonly used scales are exact. Under the assumption that needs parameters depend solely on household size and composition, the equivalence scale defined by (7) satisfies *relative equivalence scale exactness* (Lewbel 1989; Blackorby and Donaldson 1993).[8] This is due to income-ratio comparability of the welfare function (5), which requires that equality of well-being is preserved under common rescaling of the household's income (Blackorby and Donaldson 1993).

Ebert and Moyes (2003) provide a normative justification for using relative equivalence scales when the inequality concept is relative. They employ an axiomatic approach to justify the use of income-independent relative equivalence scales. By invoking the Between Type Transfer (BTT) principle and the conditions of scale invariance, income monotonicity, type monotonicity and path independence, the equivalent income function is shown to satisfy relative equivalence scale exactness. Importantly, Ebert (2010) also uses an axiomatic approach to demonstrate that relative poverty measures require use of relative equivalence scales.

2.4 A common European needs-adjusted equivalence scale

The standard approach in empirical analyses based on income after tax (cash income) is to use one common scale for all countries in comparative cross-national research on income distribution and poverty. However, to the extent that our equivalence scale estimates for public services differ across countries, the associated composite equivalence scale for extended income will also differ across countries. By contrast, if we rely on the standard assumption that the relative needs of different household types are the same in all countries, it is required to use a common equivalence scale.

As is standard for equivalence scales of cash income, we impose the conditions of unit consistency and reference independence to derive a common scale for extended income. *Unit consistency* means that the equivalence scale is invariant with respect to changes in measurement unit or currency. This condition implies that equivalence scale factors as well as measures of inequality and poverty are independent of the choice of measurement unit for a given country. *Reference independence* means that measures of (relative) inequality and poverty are independent of choice of reference household in the definition of the equivalence scale.

[8] This property is termed *independence of base* utility by Blundell and Lewbel (1991).

It follows from expression (7) that the NA scale for a given country satisfies unit consistency and reference independence. A proportional change in all needs parameters cancels out in (7), whilst a change of reference household will merely lead to a scale transformation of the country-specific NA scale. Accordingly, measures of (relative) inequality and poverty are independent of choices of measurement unit and reference household for a given country. However, these properties do not necessarily carry over to any common equivalence scale derived from the country-specific scales.

The weighted average of the country-specific equivalence scale rates emerges as a relevant candidate of a common scale. The construction of this scale requires assessment of needs-adjusted scales for each of the European countries in question. Next, the country-specific needs-adjusted scales are assigned to all households in the total population formed by all countries,[9] which requires evaluation of the needs of household h as measured by the needs parameters associated with each of the countries. To this end, it will be convenient to introduce an alternative notation to the one used in Section 2.2. Whilst NA_{hk} in Section 2.2 denotes the scale factor for a household (h) living in country k, NA_h^kdenotes the scale factor for a household (h) living in any of the countries in question when its needs are judged according to the needs parameters of country k. Thus, in the former case $h = 1,2, \ldots, H_k$, whereas in the latter case $h = 1, 2,\ldots, H$, and $H = \sum_{k=1}^{K} H_k$ is the total number of households living in the K countries. Moreover γ_{ih}^k is the needs associated with income component i for household h when needs are judged according to the needs parameters of country k, $\gamma_{+h}^k = \sum_{i=0}^{S} \gamma_{ih}^k$ is the total need of household h according to the needs parameters of country k, where $h = 1, 2,\ldots, H$. Thus, to assess the scale, factors of the common equivalence scale it is required to calculate the equivalence scale factors according to each of the K different national service standards for all households in the K countries. The common NA scale for European countries is thus defined by a weighted average of the K country-specific NA scales for every household living in these countries,

$$NA_h = \sum_{k=1}^{K} q_k NA_h^k = \sum_{k=1}^{K} q_k \frac{\gamma_{+h}^k}{\gamma_{+r}^k},\ h = 1, 2, \ldots, H, \tag{9}$$

where q_k is the weight assigned to the equivalence scale for country k. In general, the country-specific weights may depend on the needs parameters of all household types and on the choice of reference household r,

$$q_k = q_k\left(\boldsymbol{\gamma}^{\mathbf{1}}, \boldsymbol{\gamma}^{\mathbf{2}}, \ldots, \boldsymbol{\gamma}^{\mathbf{K}}, r\right), \tag{10}$$

where $\boldsymbol{\gamma}^{\mathbf{k}} = \left(\gamma_{+1}^k, \gamma_{+2}^k, \ldots, \gamma_{+H}^k\right)$ is the vector of total needs for different households derived from the spending profile in country k. Each country-specific weight is assumed to be the same for all household types. Next, we impose the following conditions on the country-specific weights:

Condition I: (*Unit Consistency*) The country-specific weights are invariant with respect to scale transformations of the needs parameters in any given country, i.e. for $\lambda_k > 0$, $k = 1, 2, \ldots, K$,

$$q_k\left(\lambda_1 \boldsymbol{\gamma}^{\mathbf{1}}, \lambda_2 \boldsymbol{\gamma}^{\mathbf{2}}, \ldots, \lambda_K \boldsymbol{\gamma}^{\mathbf{K}}, r\right) = q_k\left(\boldsymbol{\gamma}^{\mathbf{1}}, \boldsymbol{\gamma}^{\mathbf{2}}, \ldots, \boldsymbol{\gamma}^{\mathbf{K}}, r\right),\ k = 1, 2, \ldots, K.$$

[9] Household types are defined by household size and different compositions of members from different target groups. Since some of the household types do not exist in all countries, it is convenient to simulate scale rates for any household according to the needs parameters of different countries, irrespective of where the household lives. This method implies that households of equal type are given equal scale rates.

Condition I requires that the country-specific weights are not affected by a change of currency or measurement unit for any country.

Condition II: (*Reference Independence*) Change of reference household implies that the common equivalence scale will change by a constant scale parameter, i.e. for $m \neq r$ there exists a constant parameter ξ_{rm} such that

$$\sum_{k=1}^{K} q_k\left(\boldsymbol{\gamma}^{\mathbf{1}}, \boldsymbol{\gamma}^{\mathbf{2}}, \ldots, \boldsymbol{\gamma}^{\mathbf{K}}, m\right) \frac{\gamma_{+h}^{k}}{\gamma_{+m}^{k}} = \xi_{rm} \sum_{k=1}^{K} q_k\left(\boldsymbol{\gamma}^{\mathbf{1}}, \boldsymbol{\gamma}^{\mathbf{2}}, \ldots, \boldsymbol{\gamma}^{\mathbf{K}}, r\right) \frac{\gamma_{+h}^{k}}{\gamma_{+r}^{k}}.$$

Condition II assures that measurement of relative inequality and poverty in the distribution of equivalent extended income will not be affected by a change in the choice of reference group for the common NA scale.

Proposition II Let $\gamma_{++}^{k} = \sum_{h=1}^{H} \gamma_{+h}^{k}$, and let w_k, $k = 1, 2, \ldots, K$ be country-specific weights that are constant and independent of the household-specific needs parameters and the reference household. Then the following weight functions associated with the NA scale

$$q_k\left(\boldsymbol{\gamma}^{\mathbf{1}}, \boldsymbol{\gamma}^{\mathbf{2}}, \ldots, \boldsymbol{\gamma}^{\mathbf{K}}, r\right) = \frac{w_k \frac{\gamma_{+r}^{k}}{\gamma_{++}^{k}}}{\sum_{k=1}^{K} w_k \frac{\gamma_{+r}^{k}}{\gamma_{++}^{k}}}, \; k = 1, 2, \ldots, K, \tag{11}$$

satisfy conditions I and II.

The proof is given in Appendix I.

Propositions I and II show that the weighted average of the national NA scales with weights defined by (2.11) satisfies the Between Type Transfer principle as well as the conditions of reference independence and unit consistency and thus emerges as an appropriate common equivalence scale for comparison of inequality and poverty.

Choosing $w_l > 0$ and $w_k = 0$ for all $k \neq l$ means that country l is treated as a reference country, i.e. the NA scale derived for country l is applied as a common scale for a group of countries. Alternatively, we may assign equal weights to all countries or use weights that are proportional to population size. The latter method forms the basis of the empirical analysis presented in Section 3.

2.5 Assessment of the equivalence scale

Equations (7) and (9) show that the NA-scale is a function of the needs-parameters of a given household relative to the needs-parameters of a reference household. However, Eq. (8) highlights that the scale factors may vary both across different goods and services as well as between household types. Whilst the scale factors for cash income are based on the EU-scale, we make the following assumption regarding relative needs for public in-kind transfers: The mean public in-kind transfers received by different target groups reflect the value judgments regarding relative needs that are implicit in the transfer system. Thus, to quantify the needs parameters for European countries,

we use mean public spending targeted to different population subgroups defined by age and gender.[10] Mean spending per person received by different target groups, such as children and the elderly is used as indicators of the population groups' needs for childcare, education, health care and long-term care. Since the needs parameters for public services are connected to individuals, household specific needs parameters are obtained by aggregating the needs parameters of the individuals in each household.

For cash income, we use the median of the distribution of equivalent income in a given country as a basis for determining the needs parameter for the reference group. We use the EU scale to account for differences in needs of cash income for households who differ in size and composition.[11] Thus, the needs parameter of cash income for individuals of the reference household in country k is defined by

$$\gamma_{0r}^{k} = median\left(\mathbf{x}_{\mathbf{0k}}^{\mathbf{EU}}\right), \tag{12}$$

where $\mathbf{x}_{\mathbf{0k}}^{\mathbf{EU}}$ is the vector of equivalent cash incomes in country k. Note that the vector $\mathbf{x}_{\mathbf{0k}}^{\mathbf{EU}}$ includes one component for every individual in country k. Thus, $median\left(\mathbf{x}_{\mathbf{0k}}^{\mathbf{EU}}\right)$ is the median equivalent cash income in country k.[12] For households that are not of the reference type we use the chosen EU scale to assess the relative need for cash income in the following way:

$$\gamma_{0h}^{k} = \gamma_{0r}^{k} EU_{h}, \tag{13}$$

where EU_h is the scale factor for cash income pertaining to household h.

3 Empirical implementation

This section discusses methodological issues related to measurement of publicly financed in-kind transfers, where Sections 3.2 and 3.3 give an account of data and methods for valuation and allocation of public services. Section 3.4 explains the assessment of the common equivalence scale and defines three alternative measures of income. Inequality measures and poverty thresholds are defined in Section 3.5. Appendix B contains more details on the data and methods used in this study.

3.1 Population

The study relies on the EU-SILC cross-sectional data for five different reporting years, which cover the income years 2006, 2009, 2012, 2015 and 2018 and 27 EU member states as well as Iceland, Norway, and Switzerland. Six EU-SILC countries were omitted from the study due to lack of satisfactory data on public services. Table B.1 in

[10] Aaberge et al. (2010a, b, 2019) use detailed accounting data from Norwegian municipalities as a basis for estimating the NA scale for local public services. With such detailed data, they exploit minimum quantity parameters as measures of the local governments' assessment of the need of different services for different population subgroups. Detailed municipal accounting data are however not available for all European countries.

[11] The EU scale is designed to be particularly relevant for European countries, but the non-cash (NC) scale can be combined with any alternative exact relative equivalence scale for cash income.

[12] In this study the reference household type is defined by a single childless male aged 35–44 years (See Table C.1 in the Online Appendix).

Online Appendix B reports the population composition in European countries by household type.

The population in our study is classified by target groups defined by age and gender. Adults aged 18 years and above are classified by seven different age groups, where the elderly groups consist of ages 65–74 and 75 years and above. There are four age groups for school-age children, which we use to account for the allocation of government expenditures to different levels of education (primary, lower secondary and upper secondary level). Moreover, the participation rate in ECEC (Early Childhood Education and Care) varies by age. Children in pre-education age are divided into three target groups: 0 years, 1–2 years and 3 years to primary education age. Since the age intervals for attending different education levels vary between countries, the age group classification accounts for country-specific differences in the structure of the education system. Table B.2 in Appendix B shows the 14 age groups used in this study.

3.2 The value of public services

In studies of extended income, the value of public services is normally assumed to be equal to the cost of providing them (see e.g. Smeeding et al. 1993). Yet, the cost approach accounts neither for differences in quality and efficiency in the production of public services nor for possible welfare losses due to quantity constraints in the consumption of public services. Nevertheless, in line with previous studies, this study employs the cost approach as a benchmark for accounting for the distributional impact of public in-kind transfers. However, in Section 4 we perform a sensitivity analysis that relies on available estimates of public sector Technical Efficiency (TE) in developed countries.

Data on public expenditures by functions of government are made available by Eurostat, see Appendix B for details. We make use of net public expenditure, which means that households' out-of-pocket payments and other financial sources than government sources are subtracted.

Figure 1 shows public expenditure and out-of-pocket payments (household expenditure) on four welfare services by country in 2012. Expenditures and payments are normalised by the country-specific gross domestic product (GDP). Netherlands, France, Belgium, UK, Germany and the Nordic countries allocate a relatively large proportion of GDP to service provision funded by governments. Countries in Southern and Eastern Europe employ a smaller fraction of GDP on public in-kind transfers. Nordic countries display high spending on ECEC and long-term care. The Netherlands spend a high fraction on long-term care, whilst Germany, France and UK spend a high fraction of GDP on public health services. Out-of-pocket payments are relatively high in Portugal and Greece. The cross-country variation in GDP-shares and composition of public spending displayed by Fig. 1 demonstrate the importance of accounting for in-kind transfers when comparing distributions of economic well-being across European countries.

3.3 Allocation of public services

Government authorities are assumed to target public services to specific demographic groups based on an evaluation of relative needs for public services. Education services are

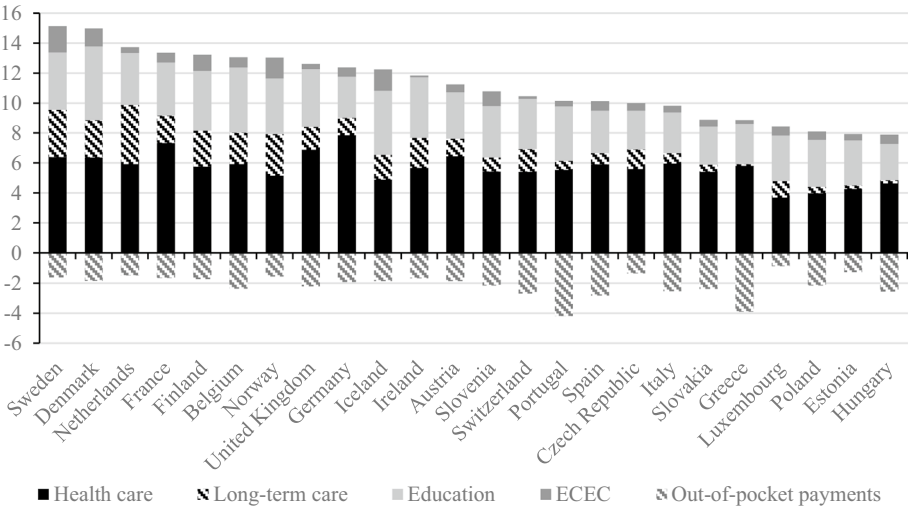

Fig. 1 Net public expenditure and out-of-pocket payments on welfare services as a proportion of GDP, 2012. Percent. *Notes*. The figure displays net public expenditure on ECEC, primary and secondary education, long-term care and health care, in percent of GDP, 2012. Out-of-pocket payments are displayed as negative numbers, which are subtracted from gross expenditure to obtain net public expenditure on welfare services. Countries are sorted by descending net public expenditure as a proportion of GDP. Source: Eurostat

provided to children because of needs for developing skills, whereas most of health care and long-term care spending is targeted to the elderly because they are more exposed to illness and disablement. Since the government decides both the selection of recipients and the type and intensity of treatment, our study accounts for the targeting policies of different governments.

3.3.1 Education and childcare services – the actual consumption approach

Two methods are used to calculate the value of public services received by individuals. Either the value is based on (i) actual consumption or (ii) expected consumption of the service. The former method is applied for the value of education and ECEC services. By dividing net public expenditure by the total number of pupils per education level (primary, lower secondary, and upper secondary), we get estimates of in-kind transfers per pupil for three education levels. For ECEC services, we also exploit information about utilisation of such services per child in the EU-SILC data. We calculate the cost per hour of ECEC services and allocate an annual value depending on average utilisation per week by age and country.[13]

[13] We do not account for higher spending per weekly hour in childcare for younger children, which might somewhat underestimate the subsidies and needs in the youngest age-group. However, relatively small subsidies for children aged 0–2 mostly reflect that they have low participation rates in ECEC services.

3.3.2 Expected spending on health and long-term care services – the insurance approach

Health and long-term care services will be treated as insurance arrangements, i.e. the values of these services are assessed on an ex-ante basis, which means that members of the target groups benefit from the expected expenditure rather than the actual expenditure on recipients. This approach has previously been applied by e.g. Smeeding et al. (1993) and Garfinkel et al. (2006). The insurance value approach is used when the type of service is considered as insurance against poor outcomes, such as poor health. Moreover, the alternative to public service provision would be to buy private insurance in the market. The expected net expenditure on health and long-term care is assumed to depend on demographic characteristics such as age and gender. Since there are different age profiles of utilisation, the allocation procedure is carried out separately for health services and long-term care.

3.4 Measurement of extended income and equivalence scales

Measurement of cash income is consistent with EU-SILC measurement of disposable income and is defined to be equal to gross income subtracted social insurance contributions and direct taxes on income and wealth. Gross income includes employee income, self-employment income and property income, which is constituted by interests, dividends and rents. Moreover, both public cash transfers and *net* inter-household transfers are treated as components of gross income.[14]

Extended income is defined by the sum of disposable income and in-kind transfers (childcare, education, health care and long-term care). To account for different needs for disposable income, we divide disposable income by the EU equivalence scale. In studies that include in-kind transfers, the common practice is to apply the same scale for extended income as is applied for cash income. As indicated in Section 2, needs for public services might differ from needs for cash income. The needs-adjusted (NA) equivalence scale used in this study accounts for differences in needs for public services and needs for cash income. Dividing extended income by the NA scale provides an income definition that allows income comparisons between people who differ with respect to needs of public services like childcare, education, health care and long-term care. Table 2 displays four alternative definitions of equivalent income that are considered in this study.

In Appendix C, we develop a simplified version of the NA scale, denoted the SNA scale. Note that the assessment of the SNA scale can be based on data for household size and composition by age groups, which makes it straightforward to apply for analyses of the distribution of extended income in any country. Appendix C reports the scale factors of the SNA scale by the number of household members of different age groups. The SNA scale is shown to be highly correlated with the NA scale and will therefore work as an appropriate approximation of the NA scale. Moreover, the scale factors of the SNA scale prove to be stable over the period 2006–2012.

We rely on the conventional practice for income distribution analyses by assuming that the household's cash income is distributed equally among household members. This

[14] Net inter-household transfers constitute the difference between such transfers received and paid. Our measure of gross income differs slightly from the definition used in EU-SILC by including gross inter-household transfers.

Table 2 Definitions of equivalent income

Income components	Equivalence scale	Equivalent income definition
Gross income – direct taxes	EU scale	Disposable income (EU)
Disposable income + public in-kind transfers	EU scale	Extended income (EU)
Disposable income + public in-kind transfers	NA scale	Extended income (NA)
Disposable income + public in-kind transfers	SNA scale	Extended income (SNA)

assumption applies also to extended income when the in-kind transfers are included in the income definition, although this does not mean that each public in-kind transfer is consumed in equal amounts by all household members.

Table A.1 in Appendix A presents country-specific relative distributions of extended income by income components in 2012. Although cash income is the dominating income component for all countries, its share of extended income differs significantly across countries.

Most of the in-kind transfers are targeted to elderly people (long-term care and health care) and families with children (ECEC and education). Therefore, the effects of in-kind transfers on income inequality will depend on the association between household disposable income and the age of household members. To illustrate this relationship, Fig. 2 displays proportions of children and elderly by quartiles of the distribution of household income for each country. In most European countries, we find that the proportion of children and elderly decreases with household income. For some countries, including Germany, the proportion of elderly is however larger in the second than the first quartile. The tendency of decreasing proportions of elderly and children with household income suggests that inequality in the distribution of extended income is lower than inequality in the distribution of disposable income.

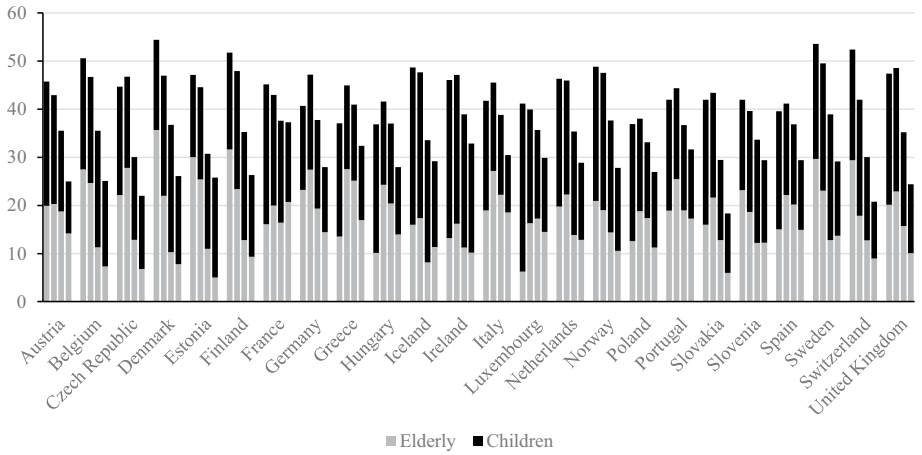

Fig. 2 Proportion of elderly and children by quartiles in the distribution of disposable income, 2012. Percent. *Notes*. The figure displays the proportion of children (aged under 18) and elderly (aged above 64) by quartiles in the distribution of disposable income in 2012. From left to right by country: quartile group 1, 2, 3, 4. Countries are ranked by alphabetical order. Source: Eurostat, authors' calculations

3.5 Measuring inequality and poverty

This section discusses and presents the methods for measuring relative inequality and poverty for country-specific distributions of economic well-being. The study of country-specific distributions can be traced back to Adam Smith who argued in *The Wealth of Nations* that poverty is the inability to afford "whatever the custom of the country renders it indecent for creditable people to be without". We refer to Brandolini (2007) and Brandolini and Rosolia (2019) for a discussion of the conceptual and empirical challenges in analysing income distribution at the supranational level as in the case of EU.

3.5.1 Inequality

Empirical analyses of income inequality are normally based on the Lorenz curve. To summarise the information content of the Lorenz curve and to achieve rankings of intersecting Lorenz curves the standard approach is to employ the Gini coefficient, which is equal to twice the area between the Lorenz curve and its equality reference. Appendix H provides results based on two complementary rank-dependent measures of inequality; one that is particularly sensitive to changes that occur in the lower part of the income distribution and the other that pays more attention to changes that take place in the upper tail of the income distribution.

3.5.2 Poverty thresholds

In most studies of poverty in developed countries, poverty is usually understood as a relative phenomenon. This perspective suggests that people compare their material situation with that of other citizens and adjust their expectations and demands for material well-being relative to the living standards of people in the same society. This study follows such reasoning and applies a relative poverty threshold to measure whether people are at-risk-of poverty or not. According to the EU method, 60% of the median equivalent income defines the poverty line; see Atkinson et al. (2002). Each country has its own poverty line for each concept of income.

4 Empirical results

This section examines the impact on inequality and poverty estimates of accounting for publicly financed in-kind transfers, while adjusting for differences in needs for such services across individuals and households. Due to missing data and other data limitations across time, we found it relevant to give priority to application of the dataset for 2012, which turned out to be the dataset with highest quality. Therefore, we show results mostly for 2012, whereas time trends in inequality and poverty are displayed in Figs. 4 and 5.

4.1 NA and SNA scale

Table C.1 in Appendix C displays the EU, NC and NA scales by household types. While the EU scale accounts for economies of scale in composite consumption of market goods and services and gives different weights to children and adults in the household, the NC

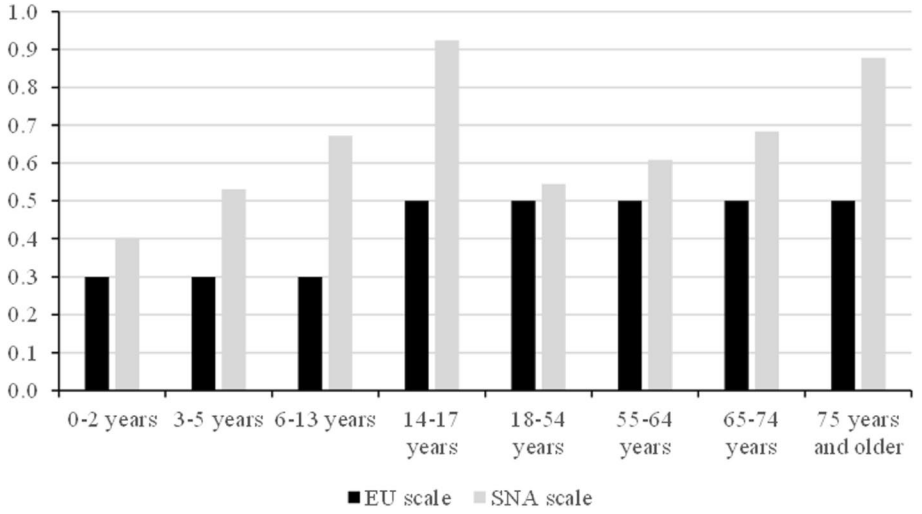

Fig. 3 EU and SNA scales for eight age groups. *Notes*. The figure displays additive weights that are assigned to individuals in different age groups. The scale factor is equal 1 for a single adult in the reference age group, which is 14 years and above for the EU scale and 18–54 years for the SNA-scale. The bars show the increase in the EU scale and the SNA scale by age group for each extra person that is added to the household. The EU scale assigns weight 1 to the household head, 0.5 to each member aged 14 and above and 0.3 to each member aged below 14. The Simplified Needs-Adjusted (SNA) scale assigns additive weights that differ by eight age groups. In our computations, the school-age groups are adjusted to account for different rules of progression by country. Source: Eurostat, authors' calculations

scale accounts for the needs of public welfare services for the household members. The NA scale combines the NC scale for public in-kind transfers with the EU scale for disposable income, while the SNA scale represents a convenient simplified version of the NA scale. Figure 3 shows estimates for the SNA scale based on data for 2012. The SNA-scale accommodates needs for childcare, primary and secondary education, health care and long-term care in addition to private consumption paid for out of cash income.

A common practice is to use the EU scale to convert household incomes for broader as well as narrower measures of income. By contrast, the present study applies a different equivalence scale when we consider inequality and poverty for extended income. To obtain information on the effect of replacing the EU scale with the SNA scale, we will also estimate inequality and poverty for extended income based on the EU scale.[15] For a given household, the change from EU scale to SNA scale represents a re-scaling of extended income that is expressed by:

$$\frac{Extended\ income}{SNA\ scale} = \frac{Extended\ income}{EU\ scale} x \frac{EU\ scale}{SNA\ scale} \tag{14}$$

where the factor of re-scaling equals the ratio of EU scale to SNA scale. Since the re-scaling factor differs across households, this transformation introduces re-ranking and relative income changes that depend on the interaction between the factor of re-scaling and households' relative positions in the distribution of extended income (EU). Intuitively,

[15] Note that the SNA scale and the NA scale produce almost identical results.

households with lower (higher) ratio of EU scale to SNA scale will tend to obtain a lower (higher) income rank when the EU scale is replaced by the SNA scale in the equivalent income definition.

Table 3 displays the ratio of the EU scale to the SNA scale for a selection of household types. For childless adults, we find that the ratio is decreasing with age, and that the relative reduction is larger for couples than for singles. For households with children, the ratio is decreasing with the number of children, and is reduced more for children in school ages compared to pre-school ages. Among the household types in Table 3, the smallest re-scaling factor is found for elderly couples aged 75 and above, and moreover for single parents with 2 children in school age. Therefore, such households may obtain relatively large reductions in income rank when the EU scale is replaced by the SNA scale. Conversely, non-elderly households without children may obtain higher income ranks because their needs are relatively low according to the SNA scale.

4.1.1 Trends in inequality and poverty

Figure 4 displays country-specific trends in the Gini-coefficient over the period 2006–2018 for both disposable income (EU) and extended income (NA). Grey bars show the inequality trends for extended income (NA), whereas the sum of grey and black bars show the inequality trends for disposable income (EU). Hence, we find that inclusion of spending on welfare services in the income definition reduces inequality estimates by 10–30%. The percentage reduction in inequality estimates tends to be larger among countries with smaller income inequality according to disposable income (EU).

We find that the levels as well as trends in income inequality varies significantly across European countries. Sweden, Denmark and Luxembourg have experienced rising inequality

Table 3 Ratio of EU scale to SNA scale by household type

Household type	Age	Singles	Couples
Childless	18–54	1.00	0.97
	55–64	0.94	0.89
	65–74	0.87	0.82
	75 +	0.75	0.68
1 child, adult(s) age 18–54	0–2	0.93	0.92
	3–school age	0.85	0.87
	School age (under 14)	0.78	0.81
	School age (over 13)	0.78	0.81
2 children, adult(s) age 18–54	0–2	0.89	0.89
	3–school age	0.77	0.80
	School age (under 14)	0.68	0.73
	School age (over 13)	0.70	0.74

Notes. The table reports the ratio of EU to SNA scale for a selection of household types. The reference household type for the SNA scale consists of single adults aged 18–54, whereas the EU scale includes all single adults in the reference household type. The age group 18–54 years includes only persons above secondary education age. For households with children, household types are constructed for the case where adults are in the reference age group (18–54). For households with 2 children, both are assumed to belong to the same age group. Source: Eurostat, authors' calculations

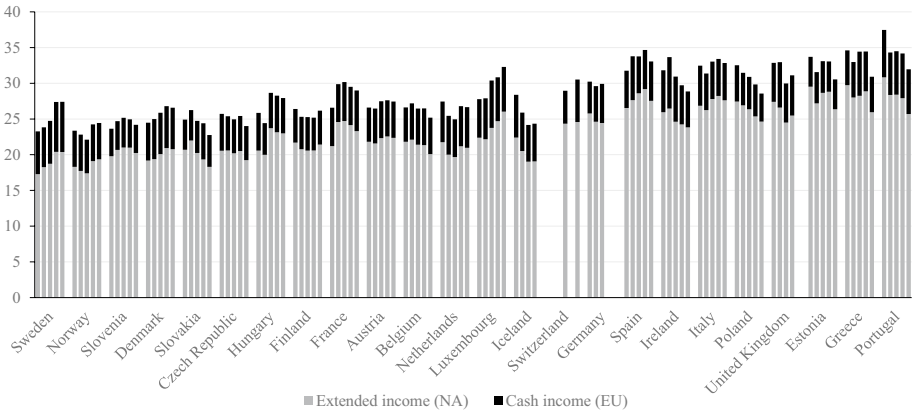

Fig. 4 Gini coefficients in European countries for disposable income (EU) and extended income (NA), 2006–2018. Percent. *Notes*. The figure displays estimates of the Gini coefficient (in percent) by country, year and income definition. From left to right by country: 2006, 2009, 2012, 2015, 2018. Inequality estimates for extended income (NA) are represented by grey bars. The sum of grey and black bars represent inequality estimates for disposable income (EU). Germany is excluded in 2015 and 2018 due to limitations on data quality. UK and Iceland are missing in 2018 because of delayed reporting of EU-SILC data. Switzerland is excluded in 2006, 2009, and 2015 due to missing data on public expenditures. Source: Eurostat, authors' calculations

over the period 2006–2018. During the same period, inequality has trended downwards in Iceland, Ireland, Poland and Portugal. In other countries, the trends are unstable or rather flat. The inequality trends are mostly parallel when comparing inequality in disposable income (EU) versus extended income (NA).

Figure 5 displays country-specific trends in poverty rates over the period 2006–2018 for disposable income (EU) and extended income (NA). Grey bars show the poverty trends for extended income (NA), whereas the sum of grey and black bars show the poverty trends for disposable income (EU). We find that inclusion of spending on welfare services in the income definition reduces poverty rate estimates by 30–60%. The percentage reduction in poverty estimates tends to be larger among countries with smaller poverty rates according to disposable income (EU), and moreover in Belgium and Ireland. We find that the levels as well as trends in poverty rates vary significantly across European countries. Sweden, Netherlands, Hungary and Luxembourg have experienced rising poverty rates over the period 2006–2018. In other countries, the poverty trend is ambiguous over time or when comparing poverty in different income definitions. For instance, Finland displays decreasing poverty in disposable income (EU) and increasing poverty in extended income (NA).

4.1.2 Inequality and poverty within groups in three different stages of the life cycle

Empirical studies typically find that incomes follow a hump-shaped profile over the life cycle. Whilst earnings are relatively low when entering the labour market, yearly incomes tend to increase up to middle ages relatively to cross-cohort averages. Later, income growth starts to stagnate and fall behind that of younger cohorts. This relative loss continues after retirement, when elderly people rely on public and private pensions as their main sources of cash income. Besides changes in average incomes by age, intra-generational income inequality may also change over the life cycle. On the one hand, income inequality could grow

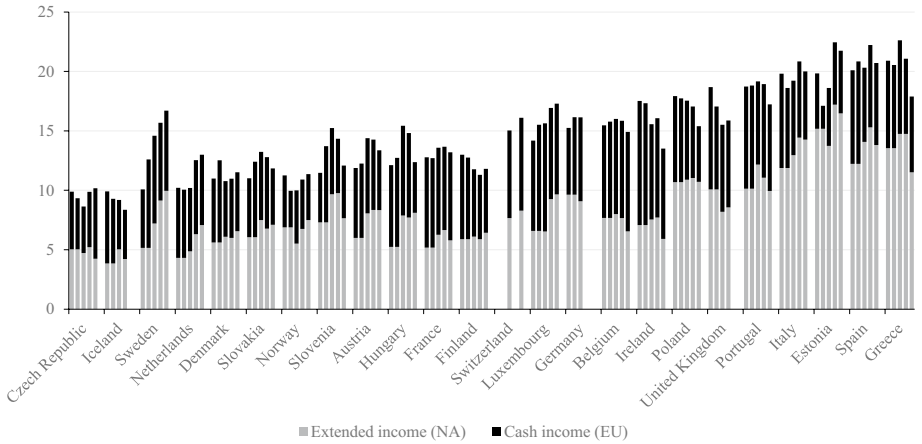

Fig. 5 Poverty rates in European countries for disposable income (EU) and extended income (NA), 2006–2018. Percent. *Notes*. The figure displays estimates of poverty rates (in percent) by country, year and income definition. From left to right by country: 2006, 2009, 2012, 2015, 2018. Poverty estimates for extended income (NA) are represented by grey bars. The sum of grey and black bars represent poverty estimates for disposable income (EU). Germany is excluded in 2015 and 2018 due to limitations on data quality. UK and Iceland are missing in 2018 because of delayed reporting of EU-SILC data. Switzerland is excluded in 2006, 2009 and 2015 due to missing data on public expenditures. Source: Eurostat, authors' calculations

with age, because of cumulative differences in the effects of luck and ability on income. On the other hand, redistribution of resources through the tax and transfer system of the welfare state might mitigate the association between age and the accumulation of income disparities.

Since the cross-sectional EU-SILC datasets do not follow individuals over time, this paper focuses on distributions of yearly snapshots of income. As a result, we are unable to distinguish between (i) inequality between individuals that persist over the life cycle, and (ii) inequality that follows mechanically when comparing individuals who are in different life-cycle stages, even when their income profiles by age are equal.

To alleviate the concern that our results may confound individual income differences with income changes over the life cycle, we will consider income distributions within three main stages of the life cycle; (1) childhood (age 0–17), (2) adulthood (age 18–64) and (3) retirement (elderly age 65 and above). Hence, while members of a given household are assumed to share incomes and obtain an equal amount of equivalent income, they are assigned to different sub-populations depending on their age. This breakdown allows us to study income distributions within the major target groups receiving public in-kind transfers, such as children and the elderly. We contrast the distributions within those directly targeted groups with working-age adults who benefit more indirectly to the extent that their households include children or elderly people.

Figure 6 shows the average Gini-coefficient in European countries within three age groups and for the total population. Moreover, inequality estimates are compared for different definitions of equivalent income. Extended income (NA) displays lower inequality estimates within each age group than disposable income (EU). Since elderly and children are major recipients of public in-kind transfers, these groups obtain large reductions in inequality estimates when including in-kind transfers in the income

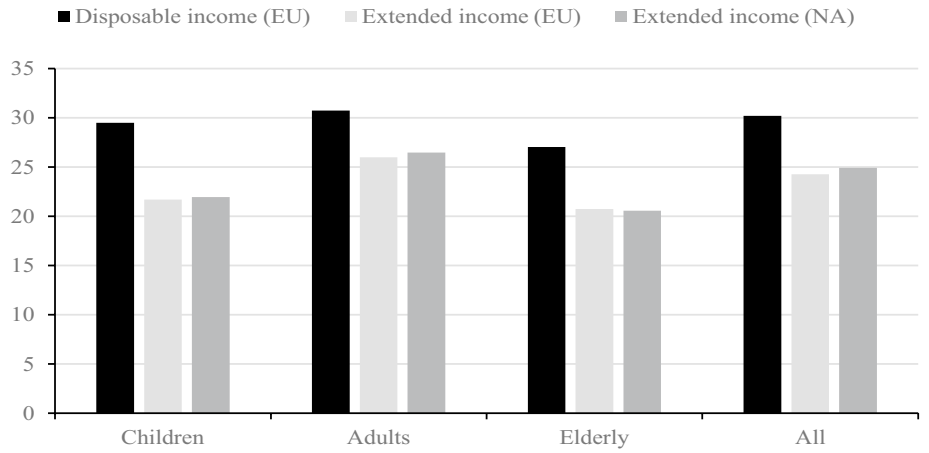

Fig. 6 Gini coefficient in population groups by life-cycle stages and income definition. Averages across European countries, 2012. Percent. *Notes*. The figure displays estimates of the Gini coefficient (in percent) within groups of children (age 0–17), adults (age 18–64) and elderly (age 65 and above), and for the total population. Inequality estimates are reported for disposable income (EU), extended income (EU) and extended income (NA). Gini coefficients are first estimated for different European countries. The figure displays weighted average estimates across European countries, where country-specific estimates are weighted by total population size. Source: Eurostat, authors' calculations

definition. By contrast, the reduction in within-group inequality is relatively small for non-elderly adults. The latter group displays higher within-group inequality than the three other household types. A reason for this is that the working-age group is relatively heterogeneous by including non-elderly singles and couples, and moreover childless adults as well as parents.

Notice that within-group inequality does not account for inequality between the different household types. As a result, within-group inequality does not add up to inequality for the total population. However, we will report group-specific poverty rates which refer to the poverty lines that are derived from the median equivalent income in the total population. Thus, the total poverty rate is a population-weighted average of poverty rates across population subgroups.

Figure 7 displays the average poverty rate in European countries within three age groups and for the total population. Extended income (NA) displays lower poverty estimates within each household type than disposable income (EU). Specifically, there is a large reduction in poverty rates in the groups of elderly and children when including in-kind transfers in the income definition. In summary, the tax and transfer systems in European countries appear to play an important role in reducing inequality and poverty, not least in groups of children and elderly who are main recipients of public in-kind transfers.

Tables A.2 and A.3 in Appendix A show inequality and poverty estimates by country and age groups in different life-cycle stages for disposable income (EU) and extended income (NA). Most European countries display relatively large reductions in estimates of inequality and poverty when moving from disposable income (EU) to extended income (NA). Yet, there is considerable variation in the percentage reduction in poverty and inequality estimates for different countries. For instance, the reduction in child poverty rates varies between 37% (Italy) and 77% (Luxembourg).

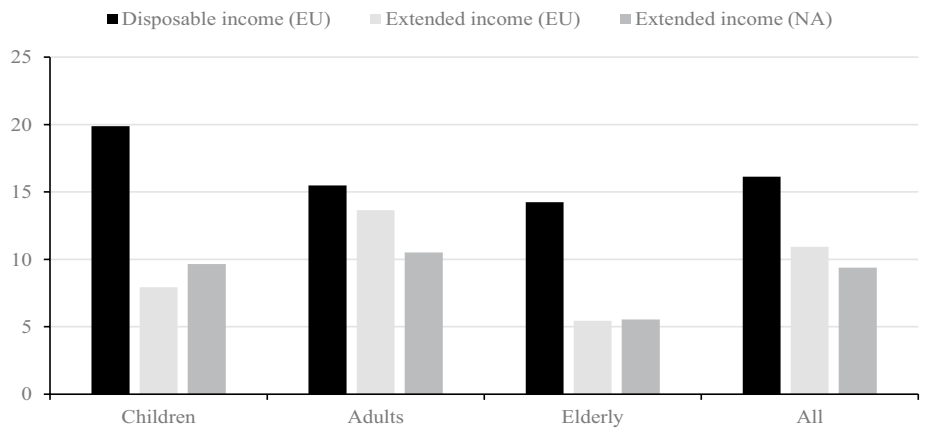

Fig. 7 European poverty rates by population groups and income definition, 2012. Percent. *Notes*. The figure displays estimates of the poverty rate (in percent) within groups of children (age 0–17), adults (age 18–64) and elderly (age 65 and above), and for the total population. Inequality estimates are reported for disposable income (EU), extended income (EU) and extended income (NA). Poverty rates are first estimated for different European countries. The figure displays weighted average estimates across European countries, where country-specific estimates are weighted by total population size. Source: Eurostat, authors' calculations

4.1.3 Sensitivity of inequality and poverty estimates to NA versus EU equivalence scale

We now investigate the sensitivity of inequality and poverty estimates to our method that makes use of the NA scale (or SNA scale) instead of the EU scale to convert household extended incomes into equivalent incomes. Figure 6 shows that when using NA scale instead of EU scale, inequality estimates for extended income are somewhat larger in the total population and within groups of children and adults, on average for European countries. Poverty estimates in Fig. 7 display an increase in poverty among children and a decrease in poverty among non-elderly adults when making use of our equivalence scale. Thus, the change in total poverty when changing equivalence scale is more muted in comparison to changes in child poverty and poverty among working-age adults. The reduced poverty rate among non-elderly adults when comparing extended income (NA) to extended income (EU) reflects that individuals in this group have relatively low needs for public in-kind transfers.

In Appendix D, we provide more comprehensive results regarding the equivalence scale sensitivity of inequality and poverty estimates as well as the extent of re-ranking when changing the equivalence scale for extended income. We find that children and elderly are typically under-represented below the median (and over-represented above the median) when using the EU scale instead of the NA scale to equivalise extended income. By making use of the EU scale instead of the NA scale, poverty estimates are significantly affected in different ways for different household types. Thus, the aggregate change in poverty tells only part of the story when evaluating the impact of our proposed NA scale in comparison to the EU scale.

4.1.4 Sensitivity of inequality and poverty estimates to public sector inefficiency

As discussed in Section 2, it would be rather demanding to account for intra-household allocative efficiency in the measurement of well-being, since this would require

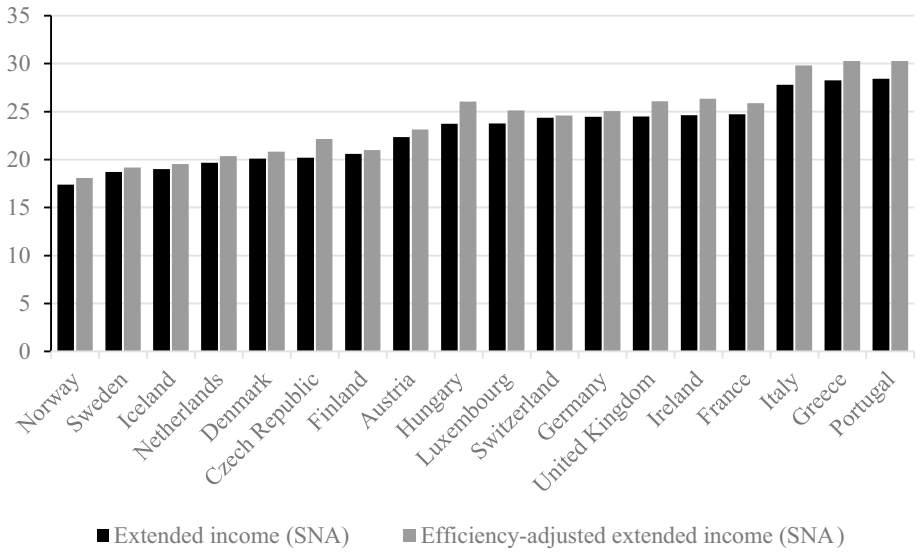

Fig. 8 Country-specific Gini coefficient estimates for extended income (baseline) versus efficiency-adjusted extended income, percent, 2012. *Notes*. The figure displays the Gini coefficient (in percent) in the distribution of extended income when the value of in-kind transfers is equal to the production cost (baseline), and when the production cost is adjusted by public sector efficiency scores in Angelopoulos et al. (2008), see their Table A.2. Belgium, Estonia, Latvia, Lithuania, Poland, Slovakia and Slovenia are not included because of missing efficiency scores. For both income measures, household extended incomes are adjusted by the NA equivalence scale. Countries are ranked from lower to higher inequality in the baseline measure. Source: Eurostat, authors' calculations

information about individuals' willingness to pay for publicly provided services. Technical efficiency is another aspect of efficiency that is just as demanding to account for, since it is often difficult to find valid output measures for public services. Nonetheless, inefficiency in public service production may reduce the value of public services received by households.

To examine the sensitivity of inequality and poverty estimates to differences in efficiency, we utilise estimates of technical efficiency of public spending reported in Angelopoulos et al. (2008). They estimate public sector efficiency in a sample of 52 countries, of which 19 countries are overlapping with our study of European countries.[16] Although such estimates of efficiency suffer from significant uncertainty, they will provide a helpful basis for illustrating how efficiency adjustment of public spending might change the picture of inequality and poverty across European countries.

As a robustness check, we multiply public expenditure by national efficiency scores for the public sector. Thus, the value of in-kind transfers is deflated in accordance with the estimated inefficiency in service production. The results of this exercise are reported in Fig. 8 and in Appendix E. According to the estimates, public sector efficiency is in the higher range in Austria, Germany, the Netherlands and in the Nordic countries, whereas efficiency is lower in Eastern and Southern European countries and in Ireland and the UK.

[16] The efficiency estimates are derived from a stochastic frontier model, where the measures of public sector output are developed by Afonso et al. (2005).

The adjustment of public expenditure by efficiency scores leads to increases in inequality and poverty estimates first and foremost in countries with a low level of efficiency. If anything, we uncover a positive association between inefficiency and inequality which implies that the difference between high and low performing countries could be understated by not accounting for cross-country variation in public sector efficiency. However, the increase in the cross-country dispersion of inequality estimates when relying on efficiency adjustment of public spending is rather modest. Appendix E provides decompositions of poverty estimates by household type. The results show that poverty estimates are more sensitive to efficiency adjustment for families with children and elderly households who live in countries with low public sector efficiency.

4.1.5 Income decomposition

Extended income can be divided into disposable cash income and the value of public services, where disposable cash income can be further divided into three main income components: market income, cash transfers and direct taxes. Whilst most cash transfers consist of public cash transfers such as pensions and social welfare benefits, such transfers also include regular inter-household transfers.

To decompose income inequality by income components, we make use of the decomposition method developed by Rao (1969), which is explained in Appendix F. The Rao method allows decomposition of the inequality share of an income component into the product of the income share and the concentration coefficient. The income share is the component's share of total income, while the concentration coefficient is equal to the conditional Gini coefficient of the component given the rank order in total income.

The decomposition results show that the tax system has an equalising effect in all countries. The equalising effect of taxes is highest in Denmark, Iceland, and the Netherlands, and lowest in Estonia, Lithuania, and Slovakia. The equalising effect of public services shows to be much smaller than the equalising effect of direct taxes. Moreover, the effects of cash transfers are neutral or weakly disequalising in most countries and strongly disequalising in Greece, Italy, Portugal and Spain.

4.1.6 Further checks of robustness

In Appendix G, the Gini-coefficient is estimated for 24 European countries using the equivalent individual rather than the individual as unit of analysis. The results show that inequality estimates based on the two different methods do not change much for given income definitions. Appendix H shows that this conclusion is also valid when the Gini-coefficient is replaced by two alternative rank-dependent measures of inequality that complement the Gini-coefficient with regard to sensitivity to transfers in the lower and upper part of the income distributions.

5 Summary and discussion

This paper introduces a method for analysing income inequality and incidence of poverty when the definition of income is extended to include the value of public services. The method establishes an exact relative and income-independent equivalence scale that ensures comparability between households with different needs. The number of household

members and their age define needs for cash income while household composition by age and gender determines the households' needs for public in-kind transfers. Importantly, the proposed needs-adjusted (NA) equivalence scale accounts for the fact that the distribution of needs for public in-kind transfers differs from that of cash incomes. Moreover, the proposed equivalence scale for public services satisfies two basic axioms, unit consistency and reference independence, which means that the equivalence scale does neither depend on the measurement unit of income nor on the choice of reference household. This is a favourable property when measures of poverty and inequality are invariant with respect to scale transformations of income.

The results from an empirical analysis based on EU-SILC data show that the level of economic inequality varies substantially across 24 European countries. Estonia, Greece, Italy, Portugal, Spain and the UK exhibit relatively high Gini coefficients, both when applying cash income and extended income as measures of economic welfare. By contrast, Austria, Belgium, the Czech Republic, the Netherlands, Slovakia, Slovenia and the Nordic countries show to have relatively low Gini coefficients. A similar pattern is found for the poverty rates. Notice, however, that there are changes in country rankings over the period 2006–2018. Inequlity and poverty have been rising in Sweden and Luxembourg and declining in Iceland, Ireland, Poland and Portugal.

Consistent with standard practice, our baseline method assumes that the value of public services is equal to the production cost. As an alternative, we make use of public sector efficiency estimates to adjust the value of in-kind transfers for technical efficiency. As expected inequality and poverty estimates for distributions of extended income increase in this case, but the ranking of countries is only slightly affected.

Appendix: Proofs

Calculations for Table 1.

The definition of the absolute equivalence scale introdced by Paulus et al. (2010) is given by

$$\frac{y_h + k_h}{e_h(abs)} = \frac{y_h}{e_h(EU)} \tag{15}$$

where y_h and k_h are the disposable income and amount of public services received by household h, while $e_h(EU)$ denotes the EU-scale and $e_h(abs)$ the new absolute scale. We use $h=0$ for the single adult and $h=1$ for the single parent. Thus, $e_0(EU)=1$ and $e_1(EU)=1.3$.

Case 1 (absolute scale).

Since we assume that Case 1 offers equal well-being for the two households, the "fixed costs" for public in-kind transfers are given by $k_0=10$ and $k_1=55$ for the absolute scale. From (15) we get the following expression for the conversition rate of the absolute scale,

$$\frac{e_1(abs)}{e_0(abs)} = \left(\frac{e_1(EU)}{e_0(EU)}\right)\left(\frac{\frac{y_1+k_1}{y_1}}{\frac{y_0+k_0}{y_0}}\right) = \left(\frac{1.3}{1}\right)\left(\frac{\frac{185}{130}}{\frac{110}{100}}\right) = \frac{185}{110} \tag{16}$$

Case 2 (relative scale).

Let $e_h(rel)$ denote the exact relative scale for extended income. By assumption, it follows from Case 1 that the relative scale factor is given by $e_1(rel)/e_0(rel)=185/110$. To maintain

equal level of welfare for the two households after adding public in-kind transfers to cash income the following condition has to be fulfilled,

$$\frac{y_1 + k_1}{e_1(rel)} = \frac{y_0 + k_0}{e_0(rel)} \tag{17}$$

which yields

$$k_1 = \frac{e_1(rel)}{e_0(rel)}\left(y_0 + k_0\right) - y_1 = \frac{185}{110}220 - 260 = 110 \tag{18}$$

Case 3 (absolute scale).
In this case, the absolute scale can be converted to a relative scale as follows:

$$\frac{e_1(abs)}{e_0(abs)} = \frac{y_1 + 55}{y_0 + 10} = \frac{260 + 55}{200 + 10} = \frac{3}{2} \tag{19}$$

where we utilise that the "fixed costs" required to satisfy needs for public services are given by 55 and 10, respectively (according to Case 1). Moreover, it is required that the amount of public services (k_1) received by household 1 should together with cash income y_1 provide the same level of welfare as $y_0 + k_0$ for household 0, which means that

$$\frac{y_1 + k_1}{e_1(abs)} = \frac{y_0 + k_0}{e_0(abs)} \tag{20}$$

which yields

$$k_1 = \frac{e_1(abs)}{e_0(abs)}\left(y_0 + k_0\right) - y_1 = \frac{3}{2}(200 + 20) - 260 = 70, \tag{21}$$

where the numbers are inserted from (19) and Case 3 in Table 1.

Proof of proposition I Inserting (2.4) into (2.1) and (2.2) yields

$$W_k\left(C_{1k}, C_{2k}, \ldots, C_{H_k k}\right) = \frac{1}{\gamma_{++k}} \sum_{h=1}^{H_k} \gamma_{+hk} V\left(\frac{C_{hk}}{\gamma_{+hk}}\right). \tag{22}$$

A marginal progressive needs-adjusted transfer from household j to household g in country k leads to the following change in welfare:

$$\begin{aligned} &\frac{W_k\left(C_{1k}, C_{2k}, \ldots, C_{gk}+\delta, \ldots, C_{jk}-\delta, \ldots, C_{H_k k}\right) - W_k\left(C_{1k}, C_{2k}, \ldots, C_{gk}, \ldots, C_{jk}, \ldots, C_{H_k k}\right)}{\delta} \\ &\underset{\delta \to 0}{\rightarrow} \frac{1}{\gamma_{++k}}\left[V'\left(\frac{C_{gk}}{\gamma_{+gk}}\right) - V'\left(\frac{C_{jk}}{\gamma_{+jk}}\right)\right]. \end{aligned} \tag{23}$$

Since the common utility function $V(\cdot)$ is strictly increasing and concave, and $(C_{gk}/\gamma_{+gk}) < (C_{jk}/\gamma_{+jk})$, it follows from (23) that W_k satisfies the BTT principle.

Proof of proposition II Since $\left(\lambda_k \gamma^k_{+r}/\lambda_k \gamma^k_{++}\right) = \left(\gamma^k_{+r}/\gamma^k_{++}\right)$ for all k we have that q_k defined by (11) satisfies Condition I. Next, inserting (11) into (9) yields the following common equivalence scale:

$$NA_h(r) = \frac{\sum_{k=1}^{K} w_k \frac{\gamma_{+h}^k}{\gamma_{++}^k}}{\sum_{k=1}^{K} w_k \frac{\gamma_{+r}^k}{\gamma_{++}^k}}, h = 1, 2, \ldots, H, \tag{24}$$

where the notation $NA_h(r)$ indicates that the NA scale might depend on the chosen reference household r. Furthermore, let

$$\xi_{rm} = \frac{\sum_{k=1}^{K} w_k \frac{\gamma_{+r}^k}{\gamma_{++}^k}}{\sum_{k=1}^{K} w_k \frac{\gamma_{+m}^k}{\gamma_{++}^k}}.$$

By changing reference household from r to m we get the following expression for the common equivalence scale:

$$NA_h(m) = \frac{\sum_{k=1}^{K} w_k \frac{\gamma_{+h}^k}{\gamma_{++}^k}}{\sum_{k=1}^{K} w_k \frac{\gamma_{+m}^k}{\gamma_{++}^k}} = \frac{\sum_{k=1}^{K} w_k \frac{\gamma_{+r}^k}{\gamma_{++}^k}}{\sum_{k=1}^{K} w_k \frac{\gamma_{+m}^k}{\gamma_{++}^k}} NA_h(r) = \xi_{rm} NA_h(r), \ h = 1, 2, \ldots, H, \tag{25}$$

which demonstrates that the NA scale with weights defined by (11) satisfies Condition II. □

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Declarations

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